Sourcing Frictions Meet Inventories
A Dynamic Ricardian Framework for the Impact of Trade Shocks

Junyuan Chen*

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Relative to long-run outcomes, short-run import responses to tariff increases may be smaller due to lower trade elasticities across sourcing countries, but larger due to inventory-driven timing adjustments in ordering. This paper develops a unifying Ricardian framework that accommodates both mechanisms while preserving tractability for solving dynamic general equilibrium responses with a realistic production network involving many industries and countries. In this model, traders set optimal intertemporal prices based on individual inventory positions and perceived future changes. Switching to optimal suppliers occurs occasionally based on expected future profits. Aggregation is simple due to a novel proportionality feature that avoids the need to track distributions of inventory levels across individual traders. A tariff shock persisting for one month, comparable in magnitude to the 2025 Liberation Day tariffs, induces a sharp decline in US imports, featuring ordering pauses among traders with sufficient inventories, and substantial cumulative welfare losses that unfold gradually over time.

The US monthly imports reached a record high of $419 billion in March 2025, just before the announcement of a package of sweeping tariff increases on April 2, a date proclaimed as "Liberation Day." Despite implementation pause and ongoing negotiations, the US imports declined substantially in May, followed by a record low of $337 billion in June since recovering from the COVID-19 disruptions.

The drastic rise and fall of US imports are unlikely to have been accompanied by a synchronous shift in expenditure on these goods among domestic consumers and producers using imported intermediates. With importers relying on inventories, the timing of the arrival of goods from abroad is detached from the timing of welfare-relevant expenditure variation. Modeling inventories is therefore essential for understanding trade impacts along global value chains.[1] Yet, practical challenges arise. Once inventories are introduced into the analysis, tackling the equilibrium responses of a model economy immediately becomes orders of magnitude more complex because of the simultaneous

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[1]Indeed, a strand of literature led by Alessandria, Kaboski and Midrigan (2010*b*) has push forward the idea that inventory adjustments stand behind short-run import dynamics. Under a closed economy, the roles of inventory movements over business cycles have drawn attention from macroeconomists decades earlier (Ramey and West 1999).

presence of a cross-sectional dimension for multilateral trade relations and an intertemporal dimension for optimal inventory management. Hitherto, quantification of dynamic trade impact when inventories are involved requires substantial simplification on the cross-sectional dimension for the sake of computational feasibility. A global economy with inventories in model is often a world with only two countries, reversing the advances in the trade literature for accommodating a realistic production network involving multiple industries and multilateral trade relations (e.g., Caliendo and Parro 2015).

This paper develops a dynamic Ricardian trade framework that overcomes the exact difficulty of preserving model details along both the cross-sectional and intertemporal dimensions, without sacrificing the feasibility of solving counterfactual outcomes under general equilibrium across monthly periods following the shocks. At the core of this model is a continuum of traders holding inventories for differentiated varieties produced from any industry where delivery is not guaranteed to be on time. These traders acquire goods from producers possibly located abroad and sell them to domestic customers at prices set on their own. Inventories arise endogenously because traders have no control on the arrival of goods from suppliers, other than adjusting the size of their orders. On the intertemporal dimension, both the size of the order placed by a trader with its supplier and the unit price it charges for the domestic customers are determined from a dynamic optimization problem taking into account the current inventory position and the perceived future evolvement in aggregate prices and quantities. On the cross-sectional dimension, a trader occasionally switches to an alternative supplier, possibly located in a different country, that supplies a variety yielding the highest expected present value of its future profit flows.

The result from enriching the model economy with such traders holding inventories is natural integration of two types of adjustment margins of trade in response to tariff shocks, differing by the relative importance depending on both the time horizons under consideration and the nature of shocks in terms of persistence and magnitude. The inventory adjustment can be viewed as the intensive margin of trade adjustments; while the gradual switching of suppliers can be viewed as the extensive margin adjustment. In response to tariff changes, traders update their optimal inventory targets. This results in a change in order size placed with the supplier. With tariff increases that are sufficiently large but temporary, a subset of traders with relatively high initial inventory stock may choose to not place an order at all in order to avoid paying the tariffs by running down existing inventories. Once the tariff rates return to normal levels, these ordering pauses are followed by larger orders for restocking. Notably, drastic changes in imports are possible without drastic changes in expenditures from households and domestic producers. Inventories play a crucial role behind such short-run phenomenon. On the other hand, in case of tariff changes that are permanent, the extensive margin adjustments take a more prominent role as time goes. The economy gradually converges to a new steady state as traders switch to new suppliers, as in Chen et al. (2025). The long-run impact of tariff changes is ultimately determined by Ricardian forces governed by the Fréchet distribution of productivity in each industry and country, as in a canonical trade framework of Eaton and Kortum (2002).

Calibration of the model parameters governing the dynamic responses of the trade flows boils down to two sets of parameters that can be estimated separately. The first set of parameters are the hazard rates governing the stochastic processes of the arrival of goods ordered by the traders. The second set consists of the elasticities of substitution across varieties, the comparative advantage parameters from the Fréchet distributions, and the probabilities of switching suppliers. Intuitively, the first set of parameters directly affects the sizes of the orders traders place by affecting how traders value the marginal inventory across time relative to the unit cost of delivered goods. The second set governs the ease of substitution across varieties from the perspective of domestic customers. These two sets of parameters jointly affect the model outcomes in general. However, taking the model as given, these two sets of parameters can be estimated separately, by exploiting tariff changes where the dominating impact on trade flows is governed by the second set of parameters. I therefore adopt the estimates from Chen et al. (2025) for the second sets of parameters and focus on estimating the first set of parameters, the hazard rates by the time since the last arrival of goods. To that end, I combine two datasets of different nature to leverage their respective strength. From a selected sample of the disaggregated monthly US trade data, I obtain the relative magnitude of hazard rates conditioning on different amount of time since the last arrival of goods. From the representative national account data, I compute monthly inventory-sales ratios among the relevant 3-digit NAICS industries. The inventory-sales ratios combined with the assumption that importers hold twice amount of inventories relative to non-importers allow me to determine the levels of hazard rates that preserve the relative magnitude from the trade data while matching the inventory-sales ratios at the industry level.

With a calibrated model covering 34 industries in 50 economies based on real-world data, I conduct counterfactual exercises inspired by the 2025 Liberation Day tariffs. These tariff changes provide a unique opportunity for demonstrating the strength of my model. First, the number of countries affected is unprecedentedly large, covering all US trade partners. The computational tractability of the model makes it possible to accommodate the country-specific tariff changes for all major economies around the world, while preserving the industry-level details. Second, for some of the countries, the proposed tariff increases are huge, with a 54% tariff rate on China.[2] It turns out that with such dramatic tariff increases, the model predicts that a fraction of traders would rather not place an order when the tariff rates just become effective. The ordering pause is a nonlinear feature of the model that appears only when the shocks are large.

To highlight the model features without dealing with additional complications in real world, I assume that the tariff rates are announced 3 months in advance, become effective for 1 month, and then immediately reverted. With such a specification of shocks, the model generates US imports from China that first increase by about 25% ahead of the implementation of tariffs and then drastically decline by roughly 75% as the 54% tariff rate becomes effective.[3] Once the tariff increases are

[2]The 54% rate includes the 20% fentanyl tariff announced in February.

[3]The implementation of a 54% tariff rate on all imports from China is hypothetical. In reality, not all goods are subject to tariffs. The 54% tariff rate on China was never actually implemented in full either, with negotiations reducing the tariff increases and implementation pauses occurring from time to time.

reverted, the imports bump up again instantly. The import responses demonstrate the anticipatory stockpiling behavior of traders and ordering pauses among traders with a sufficient level of inventories at the time when the tariff rates increase. Compared to a specification without inventories, the import responses are substantially more drastic with inventories. The loss of real wage based on aggregate labor income is much smaller on impact. However, as time goes, the cumulative loss in real wage soon exceeds that from a specification without inventories. This reflects the continuing losses during recovery time after traders are forced to depart from their long-run optimal inventory positions.

Despite a rich set of model details, the model remains computationally tractable due to a novel technique exploiting the solution properties of the inventory problems. Namely, although the inventory problems are formulated as heterogenous-agent problems involving idiosyncratic shocks to individual traders differing by both the unit cost of the unique variety traded and the inventory level in each time period, there is no need to set up a grid and solve the optimal choices of a trader given the inventory level as a state variable. Due to a proportionality property of the solutions of inventory problems, it is possible to characterize the optimal choices of individual traders on variables such as prices charged to customers and inventory targets all based on the time since receiving the last batch of goods from the supplier while holding the specific variety fixed.[4] For simplicity of terminology, I will refer to this time gap as "vintage" of goods. On the intertemporal dimension, it suffices to only compute the optimal behavior across the possible vintages for a benchmark trader with a specific variety. The optimal behavior of all other traders with the same source-destination-industry can be derived from the benchmark trader with a simple analytical formula.

On the cross-sectional dimension, aggregating the variety-level choices made across the traders requires additionally knowing the time since their last switching across suppliers. This second time gap will be referred to as "cohort" of traders. Aggregation of the vintage-specific optimal choices into aggregate variables such as the import shares requires knowing the joint discrete distribution across vintage and cohort, given the source-destination-industry. The distribution can involve many combinations of vintage and cohort. However, the evolution of the vintage and cohort across time is simple in this model. When the shocks are relatively small, the distribution across vintage and cohort is solely determined by probabilistic formulations.[5] The endogenous responses of traders only affect the distribution when the shocks are sufficiently large. In that case, a subset of traders may choose to not place an order at all and the affected vintages do not appear in the support of the distribution.

Thank to the computational tractability of the model, even with multiple industries, optimal solutions for traders importing goods from each industry and each sourcing country can be solved separately given their industry-level equilibrium prices and quantities, within an affordable amount of

[4]The productivity dispersion of the producers following the Fréchet distribution results in dispersion of productivity-adjusted unit cost, as in Eaton and Kortum (2002). The traders therefore set different optimal prices and attain different inventory levels partly due to the productivity variation across the varieties.

[5]In some sense, my approach inherits the spirit of Eaton and Kortum (2002) who promote a probabilistic formulation of comparative advantage with Fréchet distribution for the tractability of analysis.

computing time. To illustrate, with only 3 economies but 19 out of 34 industries holding inventories, the nonlinear solutions of general equilibrium outcomes to a transient shock over 100 monthly periods can be solved in around 5 seconds on a recent personal computer. When increasing the number of economies to 50 for the quantitative exercises considered in this paper where all economies except US are hit by varying levels of tariff increases simultaneously for 1 month, the general equilibrium outcomes can be solved within 1.5 hours.[6] Considering the scale of this problem, the computational tractability of the model brings a significant leap over the status quo regarding the affordable level of model details.

Lastly, it turns out that the advancement of the model on this front is not merely a matter of intellectual satisfaction, but could have tangible impact on the quantitative results reported to policymakers. By comparing the model outcomes with an alternative specification involving only 2 industries in each economy, I show that the model predictions would be exaggerated if the sectoral linkages across the 34 industries under the baseline calibration were mostly omitted. In particular, for most of the economies, the one-year cumulative real wage loss would be roughly doubled. Even in the absence of cross-industry variation in tariff changes, welfare quantification still benefits greatly from capturing details in the input-output linkages.

Relations to the Literature

This paper joins a strand of literature on the roles of inventories in shaping the responses of trade flows to shocks and their associated welfare implications. Alessandria, Kaboski and Midrigan (2010*b*) take the micro-level observation that a given variety of imported goods arrives infrequently seriously and demonstrate the relevance of inventory management for understanding trade dynamics using an (S, s)-style model where ordering of goods requires paying a fixed cost under partial equilibrium. Following this work, a series of papers have explored the roles of inventories under different context, including the 2008 Great Trade Collapse and the post-COVID supply chain disruptions (Alessandria, Kaboski and Midrigan 2010*a*, Alessandria, Kaboski and Midrigan 2013, Alessandria et al. 2023, Alessandria, Khan and Khederlarian 2024, Alessandria, Khan and Khederlarian 2025). A common limitation of this strand of work is that the quantitative exercises either focus on a partial equilibrium of trade responses or consider a world with only two economies, typically the US and the rest of the world. Despite addressing important issues arising from the global value chains, the computational complexity arising from having multiple economies is so burdensome that the researchers have chosen to abstract away from the interconnections of the global economy.

When confronting research questions of the nature that cannot be answered with a two-country world, compromises in one way or another have been made in prior work. For instance, to understand the transmission of final demand shocks along global supply chains, Ferrari (2023) takes the network structure as exogenously fixed and characterizes the inventory behavior parsimoniously by only

[6] The complexity of the problem grows quadratically with respect to the number of economies in model. With 3 economies, there are 9 bilateral relations for each industry. With 50 economies, there are 2500 bilateral relations for each industry.

relating it to sales in the next period. Recent work from Tan (2024) aims to quantify the steady-state welfare loss arising from the risk involved in global supply chains based on observed inventory intensity using a structural model. To circumvent the difficulties involved in tackling the inventory problems across 17 economies in equilibrium, the researcher introduces a notion of in-season versus off-season markets to remove the dispersion of idiosyncratic states among traders. Aside from being ad hoc, this modeling device blurs the applicability of the framework under a dynamic context for addressing the welfare impact across time following a shock.

Compared to existing work, the amount of model details feasible under my approach is exceptional. Yet, the inventory problem, which stands at the core of the framework, is still assembled from standard ingredients. In analogy to the coexistence of menu-cost-style state-dependent models and Calvo-style time-dependent models in the nominal rigidity literature, inventories can be modeled based on the fixed costs of ordering or based on the randomness in the arrival of goods.[7] I have adopted the latter approach because I consider the overall gains from doing so to be substantial, once exploiting the proportionality feature I will introduce in the paper.[8] Notably, there is no need to determine the optimal choices as functions of inventory levels. Because of that, discretization of the inventory levels with a grid is also unnecessary.[9] This is a key advantage of my model formulation over prior work in the literature that simplifies the computation tremendously. Furthermore, despite being a time-dependent model, the binary choice of whether an order is placed or not remains relevant, but in a different form of whether the optimal order size is constrained by the non-negativity constraint.[10] Some prior work has also modeled inventories in a time-dependent approach (e.g., Carreras-Valle 2024). However, to my best knowledge, no prior work has ever exploited the solution properties in my fashion to tremendously lower the barriers of incorporating inventory dynamics in a multilateral world.

Aside from the inventory problem, the model framework is tightly integrated with standard model ingredients in the trade literature. Importantly, allowing sourcing frictions as in Chen et al. (2025) provides a natural model environment where traders do not switch across suppliers instantly. The free adjustments across sourcing countries in a long-run model would be incompatible with the need of holding inventories that is induced by the lack of alternative sourcing options over the short run.[11] While staying under the Ricardian tenet, the dispersion of productivity-adjusted unit

[7]One should not view these two approaches as completely separate. If the fixed costs vary across time stochastically between zero and infinity, it becomes a Calvo-style model. Such relations are studied in more depth by macroeconomists. For example, see Auclert et al. (2023).

[8]There is a third precautionary stockout-avoidance model based on the random variation in sales that cannot be accommodated in case of stock-out. This is explored in the international context by, for example, Alessandria, Kaboski and Midrigan (2013). I do not prefer this approach as it requires the modeler to deal with the corner cases in which the goods run out of stock.

[9]Since both vintage and cohort are discrete by nature, there is no approximation errors resulting from discretizing a continuous state variable such as the inventory level.

[10]Traders are not allowed to return part of the received goods back to the suppliers. With an (s, S)-style model, the threshold that determines the binary choice of placing an order is a function of the inventory level. In contrast, with the formulation in this paper, this binary choice only depends on the vintage of the goods.

[11]On the other hand, holding the extensive margin adjustments fixed as in an Armington economy would ignore the transition of the trade responses toward the long-run trade elasticities as time goes.

cost as a motive of trade has been replaced by the dispersion of expected present value of future profit flows. Aggregation of the sourcing decisions of individual traders remains tractable due to a result from Chen et al. (2025) that generalizes the Fréchet technique from Eaton and Kortum (2002). Furthermore, due the applicability of the dynamic hat algebra in a fashion similar to Caliendo and Parro (2015), model counterfactuals can be generated with standard techniques that circumvent the need of inferring the levels of a wide range of parameters.

Broadly speaking, the paper is related to a strand of literature on the propagation of sectoral shocks along the production networks. The tariff shocks considered in the quantitative exercises can be deemed as a specific form of sectoral shocks that directly affect the pricing of economic agents at a sectoral level. By comparing the model counterfactual outcomes under the baseline specification with 19 out of 34 industries holding inventories versus an alternative specification with only 1 out of 2 industries holding inventories, I show that for many of the economies, the model responses would be exaggerated substantially after omitting the detailed sectoral linkages. This finding, which is apparently made possible by the model's capability of accommodating detailed production networks, provides an additional example of the relevance of the input-output structure of economies for shaping the impact of sectoral shocks on the aggregate economies. On this front, prior studies have explored the importance of complementarities (Baqaee and Farhi 2019; Carvalho et al. 2020), the centrality of sectors producing investment goods (vom Lehn and Winberry 2021), the role of nominal rigidity (Rubbo 2023; Minton and Wheaton 2023), the adjustment costs of changing inputs (Liu and Tsyvinski 2024) among others. My study takes the unique angle that the input-output linkages across inventory-holding industries affect the propagation of tariff shocks along the global value chains.

The paper proceeds as follows. Section I presents the model, starting from the inventory problem and moving on to the aggregation across traders by sourcing country. Section II examines the dynamic behavior of the import responses by conducting GE decompositions. Section III describes the calibration of the model for counterfactual experiments. Section IV examines the results from the model counterfactual outcomes. Section V concludes.

I. Model

In this section, I introduce and characterize the main model ingredients, highlighting the tractability of the model due to a proportionality property of the optimal solutions from the inventory problem. I first present the inventory problem faced by an individual trader, focusing on the intertemporal decisions underlying the dynamic responses in trade flows. I then embed the inventory problem in a Ricardian trade framework augmented from Chen et al. (2025) and explain the aggregation across individual trader behavior. Lastly, I define the equilibrium notion of the model economy.

A. *The Inventory Problem of Importers*

Given the supplier from which goods are acquired, the inventory problem faced by a trader is centered around maintaining an optimal level of inventories via setting the price of the variety it delivers and the size of order it places with the supplier in each period. The problem will be country-industry specific in a full model. Yet, for notational simplicity, the country and industry indices are temporarily omitted.

1. *Model Ingredients*

Time is discrete and indexed by t. Within each tradable sector of a country, there is a continuum of traders indexed by $\omega \in \Omega$ that are differentiated by the unique variety each trader supplies to the domestic economy. A trader places orders with its sole supplier that is potentially located in one of the countries, holds inventories of the goods delivered from the supplier, and sets the price of this variety for the domestic customers. Additionally, a trader distributes a dividend to the households in each period. I assume that the level of dividend varies based on the expected present value of the profit flows of the trader. Households receive a portfolio of dividends from all traders.

Timing.—The delivery of the goods from a supplier is not guaranteed at the time when a trader places its order. At the beginning of each period, the trader is informed whether its order placed in the last period is going to arrive within the current period. If the order is going to arrive, it is fulfilled by its entirety for sure. The trader pays the supplier at the current market price and counts these goods as part of the current inventories that are available for serving its own domestic customers. If the order is not going to arrive, the trader receives nothing from the supplier in the current period. The trader may freely modify the size of its order without affecting the chance of delivery in the next period.[12] After observing the current inventories, the trader sets the price it charges from its domestic customers and updates the order size for the next period. Current inventories that are not sold to customers enter the next period after depreciation.

Demand.—The varieties from all traders are delivered to sectoral good producers whose preferences across all varieties can be described with a constant elasticity of substitution (CES) aggregator. Let $q_{\omega t}$ be the amount of goods obtained from trader ω in period t. For some $\sigma > 1$, the sectoral output can be written as

$$Q_t = \left(\int_\Omega q_{\omega t}^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right)^{\frac{\sigma}{\sigma-1}} .$$

The sectoral price index can be written as

$$P_t = \left(\int_\Omega p_{\omega t}^{1-\sigma} \mathrm{d}\omega \right)^{\frac{1}{1-\sigma}} .$$

[12] The rules on how orders are placed and paid imply that there is no need to keep track of the entire history of orders placed by each trader. The "all-or-nothing" assumption captures the micro-level lumpiness of the delivery.

Each trader faces a demand schedule that can be characterized as

$$q_{\omega t} = \left(\frac{p_{\omega t}}{P_t}\right)^{-\sigma} Q_t.$$

Profit Maximization.—The trader maximizes the expected present value of the profit flows given prices and current inventories. Formally, given a supplier located in s, the dynamic optimization problem can be written as

$$\begin{aligned} V_t^s(h_{\omega t}, n_{\omega t}; \xi_{\omega t}) &= \max_{n_{\omega t+1} \geqslant 0,\ p_{\omega t}} \pi_t(p_{\omega t}) + \frac{1}{1+i_t} \mathbb{E}_t V_{t+1}^s(h_{\omega t+1}, n_{\omega t+1}; \xi_{\omega t+1}) \qquad (1) \\ \text{s.t.} \qquad \pi_t(p_{\omega t}) &= p_{\omega t} q_{\omega t} - \xi_{\omega t} \frac{c_{\omega t}}{z_s(\omega)} n_{\omega t} \\ h_{\omega t+1} &= (1-\delta)(h_{\omega t} + \xi_{\omega t} n_{\omega t} - q_{\omega t}) \\ q_{\omega t} &= \min\left\{ \left(\frac{p_{\omega t}}{P_t}\right)^{-\sigma} Q_t, \quad h_{\omega t} + \xi_{\omega t} n_{\omega t} \right\} \end{aligned}$$

where the beginning-of-period inventory level before counting potential arrival of the order is denoted as $h_{\omega t}$ and the size of order placed in the last period is written as $n_{\omega t}$. An exogenous variable $\xi_{\omega t}$ indicates whether the goods are delivered from the supplier to the trader

$$\xi_{\omega t} = \begin{cases} 1 & \text{if delivered;} \\ 0 & \text{otherwise.} \end{cases}$$

The unit cost of goods from country s is written as $c_{\omega t}$ and a productivity factor $z_s(\omega)$ adjusts the effectiveness of each unit for delivering to the domestic users. The trader takes into account the demand schedule it faces while setting optimal price $p_{\omega t}$ and discounts the expected present value in the next period with $1 + i_t$.[13] Traders face the risk for not receiving the goods ordered in the ensuing period, which is assumed to follow a Bernoulli distribution with $\chi \equiv \Pr(\xi_{\omega t} = 1)$ being an exogenous parameter independent from any choice or state of individual trader.

2. *Optimality Conditions*

Before introducing additional model ingredients, it is useful to highlight some theoretical implications of the model that can already be established.

LEMMA 1 (The Motive of Holding Inventories). *Given finite prices and aggregate quantities, a trader always chooses an optimal price $p^*_{\omega t}$ such that $q_{\omega t} < h_{\omega t} + \xi_{\omega t} n_{\omega t}$, if the probability of delivery in the ensuing period is strictly smaller than 1 ($\chi < 1$).*

Proof. Suppose that $q_{\omega t} = h_{\omega t} + \xi_{\omega t} n_{\omega t}$ under optimality. Without the arrival of order in $t + 1$ ($\xi_{\omega t+1} = 0$), the trader runs out of stock ($h_{\omega t+1} = 0$) and hence cannot deliver any good to the

[13] In the full model, the discount rate is adjusted by the changes in aggregate price index and the probability of switching to an alternative supplier.

domestic customers ($q_{\omega t+1} = 0$). In this case, the marginal revenue goes to infinity. With $\chi < 1$ and the continuity of all relevant functions, there exists some alternative $p_{\omega t} > p^*_{\omega t}$ that strictly raises the value V^s_t by letting $q_{\omega t+1}$ exceed zero under all circumstances by a sufficiently small amount. This contradicts the assumption, implying that $q_{\omega t} < h_{\omega t} + \xi_{\omega t} n_{\omega t}$ under optimality. □

Lemma 1 says that traders in this model economy never sell out their inventories by charging prices that are sufficiently high in order to avoid the scenario in which they run out of stock. This observation is useful because it guarantees that there is no need to handle the non-differentiable kinks that would otherwise be induced by the stockout constraints. With that in mind, the first order conditions of the profit maximization problem yields the following implicit characterization of the optimal price set by a trader.

Proposition 1 (Optimal Pricing). *The optimal price set by a trader satisfies*

$$p^*_{\omega t} = \begin{cases} \dfrac{\sigma}{\sigma - 1} \dfrac{c_{\omega t}}{z_s(\omega)}, & \text{if } \xi_{\omega t} = 1; \\ \chi \dfrac{\sigma}{\sigma - 1} \dfrac{c_{\omega t+1}}{z_s(\omega)} + \displaystyle\sum_{l=1}^{\infty} \kappa_{tl} \left[\chi \dfrac{\sigma}{\sigma - 1} \dfrac{c_{\omega t+l+1}}{z_s(\omega)} + p^*_{\omega t+l} \partial_h \mathcal{Q}^*_{\omega t+l} \right], & \text{if } \xi_{\omega t} = 0 \end{cases}$$

where

$$\kappa_{tl} \equiv \prod_{l'=1}^{l} \left(\frac{1-\delta}{1 + i_{t+l'-1}} \right) (1 - \chi) \quad \text{and} \quad \partial_h \mathcal{Q}^*_{\omega t+l} \equiv \frac{\partial q^*_{\omega t+l}}{\partial h_{\omega t+l}} (h^*_{\omega t+l}; \xi_{\omega t+l} = 0)$$

with $\partial_h \mathcal{Q}^*_{\omega t+l}$ denotes the derivative of the optimal supply with respect to inventory when evaluated at the optimal inventory level and the goods do not arrive in the contemporaneous period.

When the goods arrive in the current period, the optimal price set by a trader is identical to the familiar form one would expect for a static problem where the price is a markup over the marginal cost. However, the characterization for the optimal price when the goods do not arrive is complicated. It involves weighing all possibilities in the future periods. Fortunately, some further observations will allow simplifying the characterization of optimal solutions substantially. For that purpose, I introduce a compact notation $V_{t|a}$ for the value of a trader in period t that receives goods a period(s) ago but not in any period since then. In general, $V_{t|a}$ is a function of the idiosyncratic states of a trader, given aggregate prices and quantities.

Lemma 2 (Proportionality of Inventory Shadow Values). *Consider two varieties ω and ω' produced by suppliers with productivity levels z and z' respectively. The shadow values of inventory, defined as $\partial V_{t|a} / \partial h_{\omega t}$, satisfies*

$$\frac{\partial V_{t|a}}{\partial h_{\omega t}} (h^*_{\omega t|a}, n^*_{\omega t|a}; \xi_{\omega t}, z) = \frac{z'}{z} \frac{\partial V_{t|a}}{\partial h_{\omega' t}} (h^*_{\omega' t|a}, n^*_{\omega' t|a}; \xi_{\omega' t}, z') \quad \text{for all } a \geqslant 0 \tag{2}$$

where $h^*_{\omega t|a}$ and $n^*_{\omega t|a}$ denote the optimal level of inventory and optimal order size of the trader in the a period(s) since the last arrival of goods.

Proof. Suppose $a = 0$. Apply the envelope theorem with respect to h and n respectively. Combining the two resulting equations yields

$$\frac{\partial V_{t|0}}{\partial h_{\omega t}}(h^*_{\omega t|0}, n^*_{\omega t|0}; \xi_{\omega t}, z) = \frac{c_{\omega t}}{z_s(\omega)}.$$

It is clear that Equation (2) holds for $a = 0$ after taking the ratio.

Suppose Equation (2) holds for some $a \geqslant 0$. Taking the first order conditions yields

$$\frac{\partial V_{t|a}}{\partial h_{\omega t}}(h^*_{\omega t|a}, n^*_{\omega t|a}; \xi_{\omega t} = 0, z) = \chi \frac{1-\delta}{1+i}\frac{c_{\omega t+1}}{z_s(\omega)} + (1-\chi)\frac{1-\delta}{1+i}\frac{\partial V_{t+1|a+1}}{\partial h_{\omega t+1}}(h^*_{\omega t+1|a+1}, n^*_{\omega t+1|a+1}; \xi_{\omega t+1} = 0, z).$$

It follows immediately that Equation (2) holds for $a+1$ with $\xi_{\omega t+1}$ being 0 or 1. By induction, Equation (2) holds for all $a \geqslant 0$. □

Lemma 2 allows establishing the following important result.

Proposition 2 (Proportionality of Optimal Solutions by z). *Suppose that for a given productivity level z, $\{h^*_{\omega t|a}(z), q^*_{\omega t|a}(z), p^*_{\omega t|a}(z)\}$ are the optimal choices made a period(s) since the last arrival of goods. For all $a \geqslant 0$, the optimal choices under another productivity level z' satisfy*

$$p^*_{\omega t|a}(z;\xi) = \frac{z'}{z} p^*_{\omega t|a}(z';\xi); \quad q^*_{\omega t|a}(z;\xi) = \left(\frac{z'}{z}\right)^{-\sigma} q^*_{\omega t|a}(z';\xi); \quad h^*_{\omega t|a}(z;\xi) = \left(\frac{z'}{z}\right)^{-\sigma} h^*_{\omega t|a}(z';\xi).$$

Furthermore,

$$V_{t|a}(h^*_{\omega t|a}, n^*_{\omega t|a}; \xi_{\omega t}, z) = \left(\frac{z'}{z}\right)^{1-\sigma} V_{t|a}(h^*_{\omega t|a}, n^*_{\omega t|a}; \xi_{\omega t}, z').$$

Proof. For any $a \geqslant 0$, whenever $\xi_{\omega t}$ takes 1, Proposition 1 implies that $p^*_{\omega t|a}(z;\xi_{\omega t}) = \frac{z'}{z} p^*_{\omega t|a}(z';\xi_{\omega t})$. Taking the first order condition with respect to y yields

$$p^*_{\omega t|a}(z;\xi_{\omega t}) = \frac{\sigma}{\sigma-1}\frac{1-\delta}{1+i_t}\mathbb{E}\frac{\partial V_{t+1|a+1}}{\partial h_{\omega t+1}}(h^*_{\omega t+1|a+1}, n^*_{\omega t+1|a+1}; \xi_{\omega t+1}, z).$$

With Lemma 2, this implies that $p^*_{\omega t|a}(z;\xi_{\omega t}) = \frac{z'}{z} p^*_{\omega t|a}(z';\xi_{\omega t})$ holds for all a when $\xi_{\omega t}$ takes 0 as well.

Since $q^*_{\omega t|a}(z;\xi_{\omega t})$ always falls on the CES demand curve, it follows immediately that $q^*_{\omega t|a}(z;\xi_{\omega t}) = \left(\frac{z'}{z}\right)^{-\sigma} q^*_{\omega t|a}(z';\xi_{\omega t})$ for all a.

For the result on $h^*_{\omega t|a}(z;\xi)$, it suffices to show that it holds for $a = 0$. The cases for $a > 0$ follow immediately from the result on $q^*_{\omega t|a}(z)$. With the transversality condition, it must be that

$$h^*_{\omega t|0}(z;\xi_{\omega t}) = \sum_{a=0}^{\infty}(1-\delta)^{-a} q^*_{\omega t+a|a}(z;\xi_{\omega t+a} = 0).$$

That is, when the goods arrive, the inventory level reaches exactly the minimum level needed for preventing stockout in the worst scenario where the goods never arrive again in a finite number of periods. The right-hand-side of the equation reveals that the result follows from what has been shown for $q^*_{\omega t|a}(z;\xi_{\omega t})$.

Lastly, consider the implication on the expected present value of profit flows. Since $h^*_{\omega t|a}(z;\xi_{\omega t}) = (z'/z)^{-\sigma} h^*_{\omega t|a}(z';\xi_{\omega t})$ and the prices paid to the suppliers always differ by a factor of z'/z, whenever the goods arrive, the payment for the orders must be proportional by a factor of $(z'/z)^{1-\sigma}$. With the results on $p^*_{\omega t|a}(z;\xi_{\omega t})$ and $q^*_{\omega t|a}(z;\xi_{\omega t})$, it is clear that the revenue also satisfies such a proportional relation. The result on $V_{t|a}$ follows. □

Proposition 2 is a significant result for the tractability of the model, as it suggests that, holding model parameters and the number of periods since the last arrival of goods fixed, the optimal choices of all heterogenous traders in a country can be determined by solving the dynamic problem for only a single trader with an arbitrary productivity level z. Results for all other productivity levels can be obtained by simply rescaling the solutions for z. With this observation, we are ready to move on to the sourcing decisions of the traders.

B. *Frictional Sourcing Decisions*

There are N countries in the world economy and each country has I industries. The trader of variety ω in industry j of destination country d acquires the goods from a supplier with productivity $z_{sj}(\omega)$ located in the source country s. Sourcing decisions are frictional because once a trader selects the optimal supplier, it does not switch to another supplier until being hit by a Calvo-style shock, similar to Chen et al. (2025).[14] The supplier of each variety consists of competitive producers sharing the same country-specific productivity for the specific variety.

1. *Additional Model Ingredients*

Producers.—Each industry is populated by a continuum of competitive producers. A variety ω can be potentially produced in country s with a Cobb-Douglas technology that combines labor inputs with a bundle of sectoral goods. Let ℓ be the labor input and m_i be the use of sectoral goods from industry i. The production function can be written as

$$y_{sjt}(\ell, \{m_i\}; \omega) = z_{si}(\omega)\ell^{\alpha_{sj}} \prod_{i\in\mathcal{I}} m_i^{\alpha_{sij}} \quad \text{with} \quad \alpha_{sj} + \sum_{i\in\mathcal{I}} \alpha_{sij} = 1$$

where the productivity level $z_{si}(\omega)$ of each variety is independently drawn from a Fréchet distribution characterized as

$$\Pr(z_{si}(\omega) \leqslant z) = \exp(-A_{si} z^{-\theta_i}).$$

[14] Instead of introducing the sourcing friction, a simpler approach to complete the model would be based on the Armington economy, which is equivalent to setting the probability of switching supplier to zero. In that case, there is no sourcing decision to be made. On the other hand, it does not make sense to allow frictionless sourcing decisions, because that would negate the need of holding inventories.

As in Eaton and Kortum (2002), A_{si} captures the absolute advantage while θ_i captures the relative advantage.

Delivery.—Output from the producers in industry i of country s is delivered to the destination country d subject to the standard iceberg trade cost and tariffs that is captured by a factor $\tau_{sdit} \geqslant 1$. In addition, delivery of the goods is not guaranteed to happen within the same period, due to the limited capacity of transportation. The delivery of each variety is governed by the random variable ξ_{isd} introduced before. For simplicity, I assume that when a variety is not delivered in a period, production does not happen for the order placed by the affected trader. The set of varieties in production may therefore vary over time and is denoted as Ω^P_{sit} for later reference. Before adjusting the productivity of each variety, the cost of a unit of output from industry i of country s faced by traders in country d satisfies

$$c_{sdit} = \varsigma_{sdi}\tau_{sdit}w_{st}^{\alpha_{si}}\prod_{i\in\mathcal{I}} P_{sit}^{\alpha_{sij}}$$

where ς_{sdi} is a constant.

Sourcing.—As in Chen et al. (2025), in each period, a Calvo (1983)-style shock that is independent across all traders hits an individual trader in industry i with a probability ζ_i. When a trader is hit by this shock in period t, it instantly switches to a randomly selected variety ω and acquires the initial batch of goods from an optimal supplier. To be compatible with the timing assumptions made for the continuing traders introduced before, I assume that both the optimal size of the initial order and the supplier switching to are selected based on information available from the previous period $t-1$.[15] Given the initial order of optimal size $n^*_{\omega t+1}$, the optimal supplier chosen by the trader is located in one of the N countries that will generate the highest expected present value of profit flows

$$s^*_{dit}(\omega) = \arg\max_{s'\in\mathcal{N}} \mathbb{E}_{t-1}V^{s'}_{dit}(n^*_{\omega t}, n^*_{\omega t}; \xi_{\omega t} = 1, z_{s'}(\omega)). \tag{3}$$

Notice that once switching the supplier, any remaining inventory from the previous supplier is abandoned. For this reason, the initial inventory level coincides with the order size $n^*_{\omega t+1}$ in Equation (3). This is a simplification that avoids the need of taking into account all potential future suppliers while making inventory decisions with the current supplier.[16] Specifically, since the current inventories cannot be blended with goods acquired from the potential alternative suppliers in the future, the expected value of the inventories in case of future supplier switching is simply zero.[17] The discount factor for the present value V^s_{dit} therefore must be adjusted by the probability of continuing with the current supplier, which is $1-\zeta_i$. Since the timing of the supplier switching is governed by an exogenous stochastic process, only the expected present value in the future affects the sourcing

[15] The initial order is special in that its arrival is governed by the same shock that dictates the switching of supplier. This simplifies the problem without losing economic insights.

[16] An alternative approach is to assume that any remaining inventory is dumped to the consumers with a fire sale. This would add some complication without introducing additional economic insights, as the timing of the fire sale would still be governed by an exogenous process.

[17] This is consistent with the assumption that each trader only acquires goods from a single supplier. Otherwise, if goods from alternative suppliers can be freely pooled together before selling to the domestic customers, traders would simultaneously place orders across multiple suppliers to diversify the risk of non-arrival of the goods.

decisions. The history of a trader does not affect its sourcing decisions. As will be seen shortly, this simplification allows the aggregation technique underlying Chen et al. (2025) to remain applicable under the current model environment.

Households.—In each country d, a representative household consumes a bundle of final goods and supplies a fixed amount of labor in each period. The aggregate final good combines goods supplied by the sectoral good producers from each industry with Cobb-Douglas preferences

$$C_{dt} = \prod_{i \in \mathcal{I}} (C_{dit})^{\eta_{di}} \quad \text{with} \quad \sum_{i \in \mathcal{I}} \eta_{di} = 1.$$

For the goods from each industry, the household faces the same price index as the producers, which is P_{dit}. Thus, the price index for the aggregate final good can be written as

$$P^C_{dt} = \prod_{i \in \mathcal{I}} \left(\frac{P_{dit}}{\eta_{di}} \right)^{\eta_{di}}.$$

The amount of labor supplied in each country is exogenously determined as L_d. For simplicity, perfect labor mobility across industries within each country is assumed. Hence, there is only one wage level W_{dt} in each country determined by the aggregate demand of labor inputs. There is no saving in the economy. The household consumes the entirety of the income received in each period, which include the wage income and dividends from the traders. Tariff revenue from the government may be distributed to the household as a lump-sum transfer. Additionally, exogenous trade deficits may be assigned for the sake of matching trade flows in data.

2. *Aggregation*

Let $\Omega^k_{dit|a}$ be the set of varieties with the last arrival of goods happened a period(s) ago and supplier switching happened k period(s) ago. Consider a trader that just switches the supplier. For convenience, denote the expected present value of a trader with productivity 1 just switching the supplier in period t as

$$\mathcal{V}_{sdit} \equiv \mathbb{E}_{t-1} V^s_{dit|0}(n^*_{\omega t}, n^*_{\omega t}; \xi_{\omega t} = 1, z = 1).$$

Due to Proposition 2, the optimal sourcing country in Equation (3) can be rewritten as

$$s^*_{dit}(\omega) = \arg\max_{s \in \mathcal{N}} z_s(\omega)(\mathcal{V}_{sdit})^{\frac{1}{\sigma-1}}$$

where the expected present value of an inventory problem for a benchmark variety sourced from country s is scaled by the actual productivity of the variety ω. Applying the result from Chen et al. (2025), for all $\omega \in \Omega^0_{dit|0}$, the share of varieties sourced from a specific country must satisfy

$$\upsilon_{sdit} \equiv \Pr(\omega \in \Omega^0_{dit|0} : s^*_{dit}(\omega) = s) = \frac{A_{si}(\mathcal{V}_{sdit})^{\frac{\theta}{\sigma-1}}}{\Upsilon_{dit}} \tag{4}$$

where

$$\Upsilon_{dit} \equiv \sum_s A_{si} (\mathcal{V}_{sdit})^{\frac{\theta}{\sigma-1}}.$$

Equation (4) provides a compact description of the extensive margin bilateral trade relations determined one period ahead of the time among traders switching the supplier. The consideration over the entire future possibilities is captured by $\mathcal{V}_{sdit}$, the value of which only needs to be determined for a variety with a specific productivity level.

For a trader just switching the supplier ($k = 0$), the initial optimal price set for the domestic customers is a markup over the unit cost of the goods after adjusting any trade cost including tariffs. Again leveraging the results from Chen et al. (2025), within varieties obtained from just-switching traders, the expenditure share of the sectoral good producer on imports from country s can be written as

$$\lambda^0_{sdit|0} = \frac{A_{si} (\mathcal{V}_{sdit})^{\frac{\theta}{\sigma-1}-1}}{\Phi^0_{dit|0}}$$

where

$$\Phi^0_{dit|0} \equiv \sum_s A_{si} (\mathcal{V}_{sdit})^{\frac{\theta}{\sigma-1}-1}.$$

For a trader continuing staying in the market ($k > 0$), the optimal price additionally depends on the time since the last receipt of the goods (a). Proposition 2 implies that the relative intertemporal relations across prices over different a set by traders switching suppliers in the same period do not depend on z. These relative prices can therefore be characterized in terms of the optimal price $p^k_{sdit|a}$ set by a benchmark trader with some arbitrary level of z with the same k and a. Within varieties obtained from continuing traders ($k > 0$) with a given a, the expenditure share of the sectoral good producer on imports from country s satisfies

$$\lambda^k_{sdit|a} = \frac{\lambda^0_{sdit-k|0} \left(p^k_{sdit|a} / p^0_{sdit-k|0} \right)^{1-\sigma}}{\Phi^k_{dit|a}}$$

where

$$\Phi^k_{dit|a} \equiv \sum_s \lambda^0_{sdit-k|0} \left(p^k_{sdit|a} / p^0_{sdit-k|0} \right)^{1-\sigma}.$$

For aggregating expenditure shares and prices across k and a, it is useful to define the partial price index

$$P^k_{dit|a} \equiv \left(\int_{\Omega^k_{dit|a}} p^{1-\sigma}_{\omega t} \mathrm{d}\omega \right)^{\frac{1}{1-\sigma}}.$$

Similar to Chen et al. (2025), once the level of partial price index is determined for the just-switching traders, all other partial price indices are inferred based on relative relations. Let $\mu_{dit}(k, a)$ be the measure of $\Omega^k_{dit|a}$. The partial price index satisfies

$$\left(P^k_{dit|a}\right)^{1-\sigma} = \begin{cases} \gamma_i \mu_{dit}(0,0)\Phi^0_{dit|0}\Upsilon_{dit}^{\frac{\sigma-1}{\theta}-1} & \text{if } k=0,\ a=0; \\ \frac{\mu_{dit}(k,a)}{\mu_{dit-k}(0,0)}\left(P^0_{dit-k|0}\right)^{1-\sigma}\Phi^k_{dit|a} & \text{otherwise.} \end{cases}$$

The aggregate price index can therefore be written as

$$P_{dit} = P^0_{dit|0}\left[1+\sum_{k=1}^{\infty}\sum_{a=1}^{k}\frac{\mu_{dit}(k,a)}{\mu_{dit-k}(0,0)}\left(P^0_{dit-k|0}\right)^{1-\sigma}\Phi^k_{dit|a}\right]^{\frac{1}{1-\sigma}}.$$

It follows that the aggregate expenditure share of goods acquired from country s is

$$\lambda_{sdit} = \sum_{k=0}^{\infty}\sum_{a=0}^{k}\left(\frac{P^k_{dit|a}}{P_{dit}}\right)^{1-\sigma}\lambda^k_{sdit|a}.$$

Notice that, due to the presence of inventories, the expenditures of the sectoral good producers on goods acquired from different countries are not directly mapped to the imports arriving in the destination country. In this model, the imports in a period correspond to the aggregate payments for the goods just delivered to the traders ($a=0$), which include goods that are not immediately supplied to the domestic customers

$$M_{sdit} \equiv \sum_{k=0}^{\infty} M^k_{sdit} \equiv \sum_{k=0}^{\infty}\int_{\Omega^k_{sdit|0}}\frac{c_{sdit}}{z_{si}(\omega)}n_{\omega t}\mathrm{d}\omega$$

where $\Omega^k_{sdit|0}$ denotes the set of varieties within $\Omega^k_{dit|0}$ that are sourced from country s. The characterization for the imports again exploits Proposition 2. Let $n^k_{sdit-1|a}$ be the optimal order size placed one-period ahead by a benchmark trader with some arbitrary level of z from cohort k that has not received the goods for a periods by the last period. Similarly, let $h^0_{sdit|0}$ and $q^k_{sdit|0}$ be the optimal inventory level and quantity of goods supplied by the benchmark trader who just received the order placed in the last period respectively. Let $X_{dit} \equiv P_{dit}Q_{dit}$ be the total expenditure of the sectoral good producers and $\mu_{sdit}(k-1,a)$ be the measure of varieties in $\Omega^{k-1}_{sdit|a}$. The goods obtained by a cohort of traders sharing the same k may be characterized based on the relations between $q^k_{sdit|0}$ and $n^k_{sdit-1|a}$ for each a

$$M^k_{sdit} = \frac{\sigma-1}{\sigma}\mathcal{M}^k_{sdit}\lambda^k_{sdit|0}\left(\frac{P^k_{dit|0}}{P_{dit}}\right)^{1-\sigma}X_{dit}$$

where

$$\mathcal{M}^k_{sdit} = \begin{cases} \frac{h^0_{sdit|0}}{q^0_{sdit|0}} & \text{if } k=0; \\ \sum_{a=0}^{k-1}\frac{\mu_{sdit-1}(k-1,a)}{\sum_a \mu_{sdit-1}(k-1,a)}\frac{n^{k-1}_{sdit-1|a}}{q^k_{sdit|0}} & \text{if } k>0. \end{cases}$$

The import level is thus determined by a multiple over the expenditure of sectoral good producers on varieties from importers just receiving the goods. The multiple $\mathcal{M}^k_{sdit}$ is a weighted average of the ratios between the order size set in the previous period and the goods supplied in the current period, with weights determined by the distribution of importers across k in the previous period. It is now clear that the aggregate import share can be written as

$$\lambda^M_{sdit} = \frac{\sum_{k=0}^{\infty} \mathcal{M}^k_{sdit} \lambda^k_{sdit|0} \left(P^k_{dit|0}\right)^{1-\sigma}}{\Phi^M_{dit}}$$

where

$$\Phi^M_{dit} \equiv \sum_s \sum_{k=0}^{\infty} \mathcal{M}^k_{sdit} \lambda^k_{sdit|0} \left(P^k_{dit|0}\right)^{1-\sigma}.$$

C. *Equilibrium*

As a starting point, all economic agents are assumed to form perfect foresight with respect to aggregate prices and quantities.[18] Shocks to aggregate variables arrive as surprises but are fully taken into account by agents once they are announced. A general equilibrium of the model is defined as follows.

Definition 1 (General Equilibrium). A competitive equilibrium of the economy consists of paths of aggregate prices and quantities and individual policy rules of traders such that (1) the following optimization problems are solved in each period for all relevant agents

- Given $\{P_{dit}, Q_{dit}\}_t$ in home country and $\{c_{sit}\}_{st}$ for all sourcing countries, traders just switching the supplier place orders with the supplier in the sourcing countries yielding the highest expected present value of profit flows.
- Given $\{P_{dit}, Q_{dit}, \{c_{sit}\}_{st}\}_t$ and individual states, traders set optimal prices for domestic customers and optimal size of order for the next period.
- Given individual prices and available varieties, sectoral good producers aggregate varieties at the lowest cost for households and producers.
- Given sectoral price indices and wage, producers deliver output at the lowest cost.
- Given sectoral price indices and feasible income, households consume the optimal bundle of final goods.

and (2) the following market clearing conditions hold everywhere in each period

- Labor market clears in each country

$$\int_{\Omega^P_{sit}} \ell(\omega) \mathrm{d}\omega = L_d.$$

[18] Later, we consider a model extension in which importers form idiosyncratic beliefs with respect to the nature of aggregate shocks.

- Sectoral goods markets clear

$$Q_{dit} = C_{dit} + \sum_j \alpha_{sij} \int_{\Omega^P_{djt}} m_{jt}(\omega)\mathrm{d}\omega.$$

- Trade flows between producers and all importers balance

$$\int_{\Omega^P_{sit}} y_{sit}(\omega)\mathrm{d}\omega = \frac{\sigma-1}{\sigma} \sum_d \frac{\lambda^M_{sdit}}{c_{sdit}} X_{dit}.$$

Before dealing with the dynamic characteristics of the model, which is the main focus of the paper, it is useful to briefly discuss the stationary outcomes of the model. In a steady state where the levels of aggregate prices and quantities no longer vary across time, two sets of additional restrictions arise.[19] First, the quantity of goods of each industry arriving in a country must match the quantity of goods supplied to the sectoral good producers after taking into account the depreciated inventories[20]

$$M_{sdi} = \sum_{k=0}^{\infty} \sum_{a=0}^{k} \int_{\Omega^k_{sdi|a}} \left[q_\omega + \frac{\delta}{1-\delta} h_\omega \right] \mathrm{d}\omega.$$

Second, the distribution of traders by k and a is stationary within each bilateral relation. That is, $\mu_{sdi}(k, a)$ is time-invariant for all s, d and i.

Holding productivity distribution and the demand structure in each industry of every country fixed, the industry-level bilateral trade flows in a steady state can match those generated from a multisector version of Eaton and Kortum (2002), by imposing appropriate levels of bilateral trade costs and lump-sum transfers collected from some hypothetical ad valorem tariffs. Intuitively, the depreciation of the inventories would be counted as part of the bilateral trade costs under Eaton and Kortum (2002). The markups imposed by the importers for domestic customers are isomorphic to a form of ad valorem tariffs with revenue rebated to the importers.[21] One may therefore interpret the steady-state allocations in this economy as a distorted version of those in Eaton and Kortum (2002). Nonetheless, the goal of this paper is not to correct these distortions arising from the need of holding inventories and the presence of market power. We take them as given and start to examine the dynamic aspects of the economy in the next section.

II. Dynamic Responses to Trade Shocks

In this section, I provide analytical characterizations of the dynamic behavior of the economy in response to shocks to tariffs. I start from a general characterization in which all margins of adjustments are present. To justify the calibration approach to be explained in Section III, I consider

[19] The time subscripts for variables are temporarily omitted.

[20] This restriction follows from the law of motion for the inventories after aggregating across the varieties.

[21] A formal derivation for the relations of the steady-state trade flows with Eaton and Kortum (2002) is omitted as it is not critical for understanding the dynamic behavior of the economy. The argument would be similar to the proof for Proposition 1 in Chen et al. (2025).

a special case where a tariff shock is small and permanent. In Appendix A, a model extension is provided for further enriching the dynamic behavior of the model economy. Unless otherwise specified, the economy is assumed to be in a steady state prior to period 0. At the beginning of period 0, tariff changes taking into effect in period 1 or later are announced.

A. *General Characterization*

In general, tariff changes may differ by whether they are anticipated in advance (delayed implementation), the time span they are in effect, and the magnitude of the tariff rate changes. The same tariff changes may also result in different impact depending on the initial distribution of the traders over their idiosyncratic states. Given a path of the tariff changes, the dynamic responses of the trade flows in the model economy is governed by a mix of a rich set of adjustment margins among the traders along with general equilibrium (GE) feedbacks.

To guide the analysis, the response of a given bilateral trade flow at horizon $h > 0$ is decomposed into three first-order components

$$\Delta \ln M_{sdih} \approx \text{Intensive Margins} + \text{Extensive Margins} + \text{GE Responses}.$$

Here, the intensive margin adjustments refer to the responses of imports arising from the changes in order sizes across all traders placing an order in the previous period. The extensive margin adjustments are the changes in sourcing decisions among supplier-switching traders. The GE responses are the impact due to changes in aggregate prices around the world. Relative to Chen et al. (2025), the bulk of the innovation of this model arises from the intensive margin adjustments. Below, I focus on the intensive margin adjustments in detail. The extensive margin adjustments work similarly as in Chen et al. (2025). I do not analytically characterize the GE responses and only examine that quantitatively later in Section IV.

Holding the set of traders placing a positive order in the previous period unchanged, the intensive margin adjustments can be split into two parts

$$\text{Intensive Margins} = \underbrace{\sum_{k=0}^{\infty} \int_{\Omega^k_{sdih|0}} z_s^{\sigma} \Delta h^{*}_{sdih|0} \mathrm{d}\omega}_{\text{Changes in inventory targets}} + \underbrace{\sum_{k=0}^{\infty} \sum_{a=0}^{k} \chi_a \int_{\Omega^k_{sdih-1|a}} -z_s^{\sigma} \Delta h^{k}_{sdih|a} \mathbf{1}\left(n^k_{sdih|a} > 0\right) \mathrm{d}\omega}_{\text{Changes in existing inventories before restocking}} \tag{5}$$

reflecting the changes in the inventory targets reached by traders just receiving goods at horizon h and the changes in inventory levels before counting the newly arrived goods among non-switching traders placing a positive order in the previous period.[22] Notice that the two parts of the intensive margin adjustments are determined based on very different factors. Changes in inventory targets are forward-looking in that they are the minimum amount of goods required for sustaining positive sales in all circumstances in the future. Changes in existing inventories before restocking are

[22] The second component does not involve just-switching traders because they always place the initial order starting from zero existing inventory.

backward-looking as they result from the orders placed in the past and how fast the traders deplete the inventories prior to the adjustments. Put together, the intensive margin adjustments are jointly determined by how traders perceive the future changes given the current supplier and what traders have already experienced from changes in the past.

Notably, Equation (5) is not a full characterization as it requires fixing the set of traders placing a positive order. Indeed, the trade elasticities are not continuous in this model, because the set of traders placing a positive order in the previous period may change in a discrete manner following a shock. This observation is important enough to be highlighted below.

PROPOSITION 3 (Jumps in Trade Elasticities). *Ignoring GE responses by fixing aggregate prices fixed, the trade elasticity at each horizon is a discontinuous function of the tariff rate changes with countably many discrete jumps.*

Proof. Notice that the inequality constraint $n_{\omega t+1} \geqslant 0$ in Equation (1) may become binding for subsets of the traders (with strictly positive measure over the variety space) at certain levels of positive tariff changes as the shocks become larger and larger. More precisely, for all $a \geqslant 0$, the shadow value of the inventories satisfies the following relations under optimality

$$\frac{\partial V^s_{t|a}}{\partial h_{\omega t}} = \begin{cases} \frac{(1-\zeta_i)(1-\delta)}{1+i_t}\left[\chi_a \frac{c_{\omega t+1}}{z_s(\omega)} + (1-\chi_a)\frac{\partial V^s_{t+1|a+1}}{\partial h_{\omega t+1}}\right] & \text{if } n_{\omega t+1} > 0; \\ \frac{(1-\zeta_i)(1-\delta)}{1+i_t}\frac{\partial V^s_{t+1|a+1}}{\partial h_{\omega t+1}} & \text{if } n_{\omega t+1} = 0. \end{cases}$$

Intuitively, when the marginal value of restocking $\chi_a \frac{c_{\omega t+1}}{z_s(\omega)}$ is higher than the value of not receiving the goods, the trader would rather not place an order. Furthermore, despite the contingency depending on whether $n_{\omega t+1}$ is strictly positive, it is not hard to see that Proposition 2 continues to hold. Put together, these observations imply that for some cutoff level of the tariff rate changes $\Delta\tau^*_a$, all traders receiving goods a period(s) ago switch between placing or not placing a strictly positive order simultaneously, no matter the productivity of their individual varieties. This result holds no matter when the sourcing decisions are made (k). Hence, when considering traders forming the trade relations in different periods, each subset of these trades has a cutoff $\Delta\tau^{k*}_a$ additionally indexed by k.[23]

Now return to Equation (5). The set of cutoffs $\{\Delta\tau^{k*}_a\}_{0\leqslant a\leqslant k, k\geqslant 0}$ form a partition of the possible levels of tariff changes. As $\Delta\tau$ goes from 0 to infinity, changes in $\ln M_{sdih}$ is continuous between each pair of adjacent cutoffs due to the continuous nature of the solutions from the optimization problems. Yet, as passing a threshold $\Delta\tau^{k*}_a$, the simultaneous switching to zero order among the subset of traders of variety $\Omega^k_{sdih|a}$ results in a kink in the intensive margin adjustments. It follows that there are countably many discrete jumps in the derivative, which is exactly the trade elasticity as a function of the shock magnitude. □

[23] The only exception is the initial orders placed by traders who are switching the supplier. The order-placing in this circumstance is special as the trader is starting with zero inventory.

Proposition 3 emphasizes that the trade elasticities additionally vary by the magnitude of the shocks. It highlights possibilities of trade responses that are not seen in a standard trade model without inventory. Intuitively, inventories can be thought of as a secondary source for fulfilling the domestic sales. The choice between placing orders for potential arrival of additional goods and only relying on existing inventories really hinges on which one results in larger expected present value for the trader. With large shocks, trade elasticities can look differently as more traders choose to run down existing inventories without placing an order.

The above discussions accompanied by the sluggish extensive margin adjustments considered in Chen et al. (2025) completes the analytical characterization. To help forming intuition on the dynamic responses of the trade flows in this model, I proceed with additional discussions with a special case of the tariff shocks.

B. *Special Case: Small Permanent Shocks*

In a steady state, all traders place orders of strictly positive size. In other words, no trader would forgo an opportunity of stocking goods into the inventories. With this observation in mind, it is not hard to see that if the magnitude of shocks are sufficiently small, the same would happen as the economy moves to a new steady state. In this scenario, the economy is experiencing a small shock. Assuming that the shock is a permanent tariff change taking into effect since period 1, we have the following result similar to Proposition 2 in Chen et al. (2025).

Proposition 4 (Partial Equilibrium Trade Elasticity for Small Shocks). *Suppose that the economy is in steady state prior to the permanent shock announced in period 0. Holding aggregate price indices and the aggregate demand from the sectoral good producers unchanged, for $h \geqslant 1$, the partial equilibrium elasticity of the horizon-h bilateral trade flows with respect to a small permanent tariff shock announced at time $t = 0$ and taking into effect at time $t = 1$ is*

$$\varepsilon^h_{sdi}(\Delta\tau) \equiv \left.\frac{\partial \ln M_{sdih}}{\partial \ln \tau_{sdi1}}\right|_{\Phi^M_{dit}} (\Delta\tau) = -\theta_i \left[1 - (1-\zeta_i)^h\right] - (\sigma_i - 1)(1-\zeta_i)^h$$

when the magnitude of the tariff change $\Delta\tau$ is sufficiently small.

Except some variation arising from the need of placing orders one period ahead, the characterization above for the trade elasticity matches the one from Chen et al. (2025) that does not involve inventories. This suggests that the horizon-specific characterization of trade elasticities continues to hold in the current model when tariff shocks are small, but not in general due to Proposition 3.

Yet, I stress that Proposition 4 is a result based on partial equilibrium. Under general equilibrium, even with small tariff shocks, there is a difference regarding the dynamic impact to the economy relative to Chen et al. (2025) due to how the tariff impact is transmitted to the rest of the economy. That is, the aggregate prices and quantities in this model are affected differently because of the optimal inventory management of the traders. To see this, it is useful to compare the pricing rule of the model with Chen et al. (2025) where all prices share the same markup determined by the elasticity of substitution. There, without inventories, the prices for all varieties faced by the domestic users share the same markup over the unit cost. Here, with the optimal pricing arising from the inventory problem, traders charge varying markups that additionally depend on their idiosyncratic inventory levels. The insight from the comparison is important enough for it to be stated formally.

PROPOSITION 5 (Partial Tariff Passthrough). *With a permanent tariff shock that raises the unit cost of goods from country s faced by traders in country d by a factor of $\tau > 1$ for all periods $t > 0$, the partial price indices for all traders that have not received the goods again since $t = 1$ must satisfy*

$$\left.\frac{P^k_{dit|a}}{P^k_{di0|a-t}}\right|_{\{c_{isdt}=\tau c_{isd0}\}_{t\geqslant 0}} < \tau$$

even if the path of unit cost $\{c_{isdt}\}_{t\geqslant 1}$ after the tariff increase is externally fixed to be a multiple of the initial level τc_{isd0}.[24]

Proof. Following the permanent shock, the optimal rules of all traders immediately switch to the new ones under the shock. Yet, the distribution of traders over the idiosyncratic states is affected by where they start with from the original steady state before $t = 0$. Notice that the tariff shock is isomorphic to a shock that scales down the productivity of all varieties from country s in the view of country d. Lemma 2 then implies that following the shock, the optimal inventory levels of all traders in the new steady state will be lower. It follows that given the initial distribution of inventories from the original steady state, the idiosyncratic levels of inventories held by all traders are exceeding the levels in the new steady state. Furthermore, holding $\{c_{isdt}\}_{t\geqslant 1}$ as time-invariant multiples of $\{c_{isdt}\}_{t\geqslant 1}$ implies that under the new steady state, all prices are multiples of the prices in the original steady state by the common factor τ. Since traders are holding more inventories than the levels in the new steady state until they reset their inventory levels to the new optimal level under the shock, it must be that the prices set by such traders are lower than the ones in the new steady state. The result on partial price indices follows. □

Proposition 5 says that, compared to a model where prices share the same markup, the domestic customers experience a smaller price increase compared to the magnitude of the tariff change due to the optimal inventory management. The smaller price increase, at least over the short-run, implies a smaller drop of expenditure share on goods imported from the affected country. This in turn slows down the process of switching to suppliers in alternative countries among traders for newly entered varieties. Therefore, due to the partial passthrough of tariff increase to aggregate price indices, the responses of aggregate trade flows are even smaller on impact, compared to the benchmark model of Chen et al. (2025) without inventories.

[24]The purpose of holding unit costs externally is to exclude the general equilibrium effect on foreign unit costs due to decrease of wages in country s.

III. Calibration

For running counterfactual experiments, the model is calibrated to an initial steady state based on recent data. The required parameters can be classified into 4 sets based on the data sources supporting the calibration. For most of this section, I focus on the hazard rates governing the arrival of goods. Calibration of the other parameters is only briefly discussed towards the end.

A. *Hazard Rates of Goods Arrival*

The model requires estimates of the hazard rates of goods arrival conditioning on the vintage of the existing inventories. At the finest level for the baseline model, these parameters additionally vary by the industry of the goods traded and the bilateral relation, and hence could be denoted as $\{\chi_{aisd}\}$. These are the main additional parameters after introducing inventories into the framework. Taking the model as given, the hazard rates directly affect the extent to which traders rely on inventories for supplying their domestic customers. With low hazard rates, traders place larger orders and hold more inventories. In response to tariff increases that are temporary, they are more likely to avoid tariff payments by not placing an order.[25]

To attain reasonable estimates, at least for traders acquiring goods from US., I leverage two data sources of different nature jointly. First, I collect the monthly US import and export data from 2022 to 2024 at one of the lowest aggregation levels the US Census Bureau publicly provides. These data disaggregate trade volumes by 10-digit HS code, US port district, source/destination country and dutiability simultaneously.[26] These trade data do not provide information on domestic traders that do not import from abroad. Second, I bring in the monthly data on real inventory stock and real sales from the NIPA underlying detail tables published by the US Bureau of Economic Analysis (BEA). These data provide estimates by 3-digit NAICS industries within the manufacturing, wholesale and retail sectors, but are intended to only cover goods produced in the US.

Except the apparently different data coverage, these two datasets have different strength. For the trade data, the low aggregation level makes it possible to infer the approximate micro-level delivery frequencies faced by individual traders. On the other hand, the NIPA data are nationally representative for the US and provide direct estimates for the levels of inventory stocks. To take advantage of the strength of both these data sources, I first infer the relative magnitude of hazard rates for traders holding inventories of different vintages from the trade data. I then determine the levels of the hazard rates by relating the model-implied average inventory-sales ratios to the counterparts from the NIPA data while assuming that importers have inventory-sales ratios that are twice as high as the ratios for domestic traders. The ratio of 2 for inventory-sales ratios between importers and non-importers is a popular estimate adopted in prior work (e.g., Alessandria et al. 2023).

[25] Since the model assumes that traders can freely adjust their order size in each month, the information on when an order is placed is irrelevant in this model. All that matters is the frequency at which goods arrive.

[26] As one of the data sources for their empirical evidence on the lumpiness of trade, Alessandria, Kaboski and Midrigan (2010*b*) have examined a historical version of the US Census trade data similar to what I collect.

To elaborate on the calibration procedure, I start from the construction of the trade sample. For the sake of calibration, I assume that a variety in model is defined at the aggregation level that the trade data provide. Despite the large amount of details provided, it is impossible to distinguish the trade volumes by individual importer/exporter. To approximate the possible behavior of an individual trader, I make use of a variable called "card count" that records the number of import/export activities happened for each variety in a month. A variety is preserved in sample only if its maximum card count has never exceeded 4 in any month during the 3 years.[27] The rationale is that for most of the goods crossing country boarders, it is unlikely for a single importer/exporter to acquire/deliver a specific variety more frequently than a weekly basis. It is however totally possible that multiple traders stand behind the trade flows reported in data, despite the sample selection. I therefore deem the hazard rates estimated from the trade sample as approximate empirical counterparts of the hazard rates in model. In particular, I only make use of the relative magnitude across vintages from these estimates to be conservative.

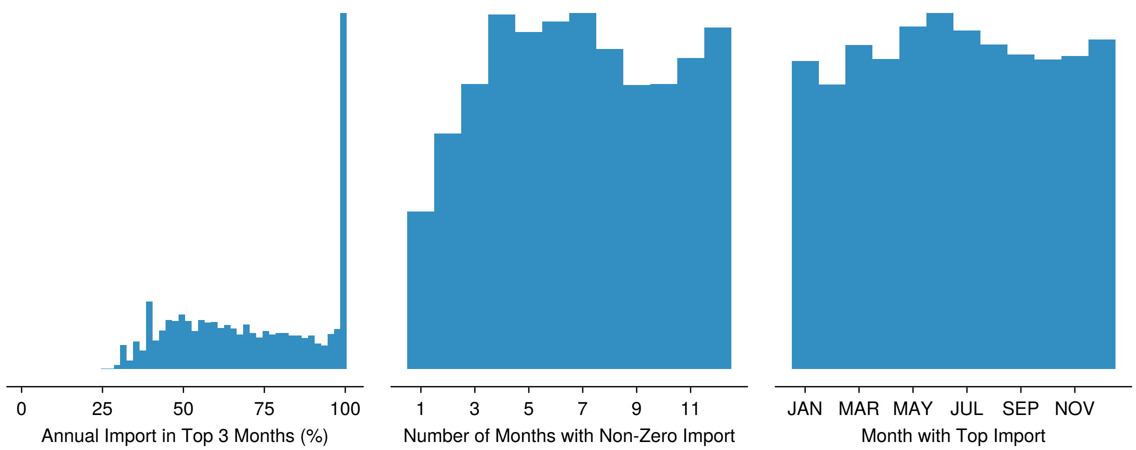

FIGURE 1: Histograms for the Selected US Import Sample

Note: The sample is summarized at the level of 10-digit HS code by US port district by sourcing country by dutiability within each year from 2022 to 2024. Data are obtained from the US Census Bureau and selected based on the procedure described in the text.

Figure 1 summarizes some aspects of the selected US import sample.[28] Echoing the findings from Alessandria, Kaboski and Midrigan (2010*b*), individual traders are likely to receive goods only occasionally in large batches. The timing of when the bulk majority of goods arrive for an individual trader within a year also seems to be mostly irrelevant to seasonality. From this sample, I track the binary outcomes on whether positive trade flows are reported for each variety following the last arrival of the goods. These binary outcomes, accompanied by the time gap since the last arrival, allow me to compute the hazard rates by taking the average within each cell partitioned based on

[27] I additionally require that positive trade flows of each variety must have appeared in data for at least once in each of the 3 years and the HS code can be matched to one of the NAICS industry. This helps avoid varieties that are not actively traded or are not associated with a specific industry.

[28] The histograms for the selected US export sample look similar and are provided in Appendix B.

the trade partner and associated industry of the goods. The same procedure is repeated for the US export data. All resulting estimates are summarized in Figure 2. It turns out that, within the selected sample, many goods are delivered again within the quarter, but some are only delivered over longer periods.

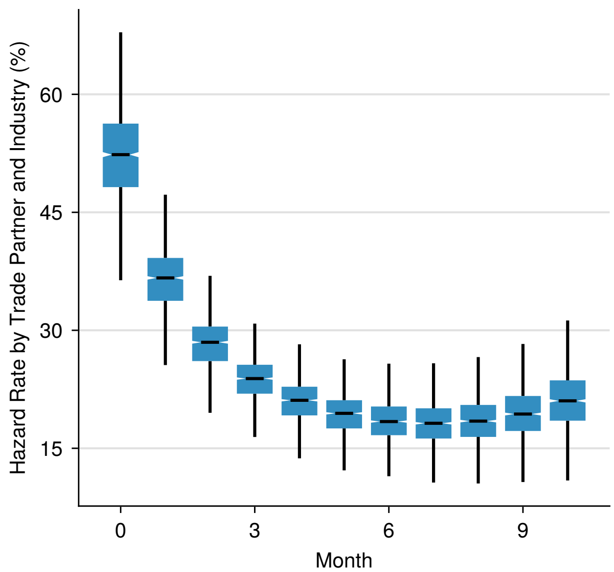

FIGURE 2: Box Plots of the Estimated Hazard Rates by Trade Partner and Industry

Note: The hazard rates are estimated separately for each observed combination of source/destination and industry. Countries/regions and industries are at levels matching those selected for the counterfactual experiments. Each box represents the interquartile range with the medium dividing the box into two parts. The vertical bars show the full range of the estimates.

With these preliminary estimates, I preserve their relative magnitude across vintages but determine the levels fed into the model based on inventory-sales ratios computed from the relevant 3-digit NAICS industries from the NIPA data. For each industry, I combine monthly inventory and sales data from both the manufacturing industries and possibly the relevant wholesale or retail industries.[29] The average monthly inventory-sales ratios from 2022 to 2024 are taken as empirical targets. I then rescale the hazard rates from the trade data. To construct the hazard rates for the domestic US traders supplying goods from a specific industry, I first obtain the average hazard rates from the trade data within the industry across all foreign trade partners. I then seek a scaling factor such that the model-implied average inventory-sales ratio across all such traders match the counterpart from the NIPA data.[30] This gives me the hazard rates specified for the domestic traders in model. To obtain the hazard rates for the importers, I adjust the hazard rates from the trade data in a

[29]This addresses the possibility that a substantial fraction of inventories for imported goods may be held by wholesalers and retailers. See Table B.1 in Appendix B for the NIPA series matched to each model industry.

[30]Precisely, the model average inventory-sales ratio is defined as the ratio of $\sum_{k=0}^{\infty}\sum_{a=0}^{k}\mu_{sdi}(k,a)h_{sdi}(k,a)$ over $\sum_{k=0}^{\infty}\sum_{a=0}^{k}\mu_{sdi}(k,a)q_{sdi}(k,a)$ where the former is the total inventory level of the benchmark trader across distribution by cohort and vintage and the latter is for sales.

similar manner so that the model-implied average inventory-sales ratios are twice as high as the NIPA counterparts.

TABLE 1: Targeted Inventory-Sales Ratios from Data and Model

Industry	BEA NIPA Average	Inventory-Sales Ratio in Model: US Domestic Traders	Inventory-Sales Ratio in Model: US Importers
Agriculture	1.94	1.94	3.50
Mining	2.46	2.46	4.53
Food	0.98	0.98	1.97
Textiles	2.28	2.28	3.51
Wood	1.41	1.41	2.81
Paper and printing	1.60	1.60	3.20
Coke and petroleum	0.90	0.90	1.80
Chemicals	2.05	2.05	4.11
Pharmaceuticals	1.35	1.35	2.70
Rubber and plastics	1.93	1.93	3.87
Non-metallic minerals	2.06	2.06	4.12
Basic metals	2.30	2.30	4.59
Fabricated metals	2.37	2.37	4.74
Computer	1.92	1.92	3.85
Electrical equipment	1.29	1.29	2.58
Machinery	2.76	2.76	5.53
Motor vehicles	1.74	1.74	3.48
Other transport equip.	4.61	4.61	8.88
Furniture and others	2.02	2.02	4.04

Notes: In the model, there are 19 industries among a total of 34 industries that hold inventories and are exposed to tariff changes. These industries are classified based on the ISIC classification adopted by the OECD ICIO table. The BEA NIPA average inventory-sales ratios are constructed by combining the monthly data on real inventory stocks and real sales for the relevant 3-digit NAICS industries from 2022–2024. Whenever possible, inventories and sales data for the relevant wholesale or retail sectors are included along with the manufacturing industry. The names of the industries listed in the table have been simplified from the names in the original classification system for ease of exposition.

The industry-specific inventory-sales ratios from NIPA data used as empirical targets and the attained ratios in model are reported in Table 1. As long as one accepts the approximate relation that importers hold double amount of inventories compared with non-importers, the calibrated hazard rates have respected both the relative magnitude across vintages within each bilateral trade relation from the trade sample and the domestic inventory levels in US. Still, the model requires estimates for traders in other countries. Future work may bring in additional data sources covering trade flows not involving the US. However, as a starting point, I simply impute the hazard rates for traders in other countries using the industry-specific averages among the US traders.[31]

[31] As a technical note, for the quantitative exercises, additional hazard rates going beyond the 11th month up to the end of the second year are filled in so that the hazard rates gradually reach 1. This ensures that no traders wait for restocks forever, although more than 95% of the traders in model already receive restocks within a year.

B. *Other Parameters*

Calibration of the other three sets of parameters follows Chen et al. (2025) closely.

First, from the OECD Inter-Country Input-Output (ICIO) table of 2019 (Yamano et al. 2023),[32] I directly obtain the parameters governing the household expenditure shares across goods from each industry, the expenditure shares across intermediate inputs from each industry of producers in a given industry, and the bilateral import shares across trade partners for each economy. The level of final use of goods from each industry in each economy is targeted exactly.

Second, based on the discussion on the impact of small permanent shocks from Section II, I deem it appropriate to reuse the elasticity estimates $(\sigma_i, \theta_i, \zeta_i)$ from Chen et al. (2025). These estimates are obtained from nonlinear GMM estimation based on a structural equation for the horizon-specific trade elasticities.[33] Plausibly exogenous small tariff changes from Boehm, Levchenko and Pandalai-Nayar (2023) are used as instruments to deal with the endogeneity concern.

Lastly, I directly specify some parameters based on convention. The discount rate of future profits before adjusting the probability of switching supplier and aggregate price changes is set to be $0.95^{\frac{1}{12}}$. The depreciation rate for inventories is set to be 0.025 across all industries following Alessandria, Kaboski and Midrigan (2010*b*).

IV. Quantitative Applications

In this section, I conduct counterfactual experiments highlighting the key features of the model.

A. *The Liberation Day Tariffs*

On April 2 2025, following the declaration of national emergency, President Trump announced tariff increases on all US trader partners that would be implemented starting from April 9. Under the initial plan, the US will impose a 34% tariff rate on China in addition to the 20% fentanyl-related tariff announced in February, resulting in a drastic 54% tariff hike. Yet, on April 9, an immediate 90-day suspension of the tariff implementation for all countries except China was announced, in response to the announcements of retaliatory tariffs from China on US. Since May 14, the tariff increase on China was temporarily reduced to 10% for 90 days until August 12, as a suspension of the implementation of new tariff rates under negotiations. On August 11, the suspension was extended for another 90-day period.[34]

Despite suspensions in tariff implementation, there is little doubt that anticipation and turmoil related to the Liberation Day tariffs have had substantial impact on the US imports from China. As depicted in Figure 3, the sharp decline in US import surrounding the Liberation Day has been the only episode over the last decade that is comparable to the COVID-19 disruption in terms of the

[32] The ICIO table is available up to 2020 at the time of writing. However, I use the 2019 table to avoid the unusual circumstances due to the COVID-19.

[33] The estimates are reported in Table B.2 from Appendix B.

[34] Exact details of the policy changes can be found from White House announcements made on the respective dates.

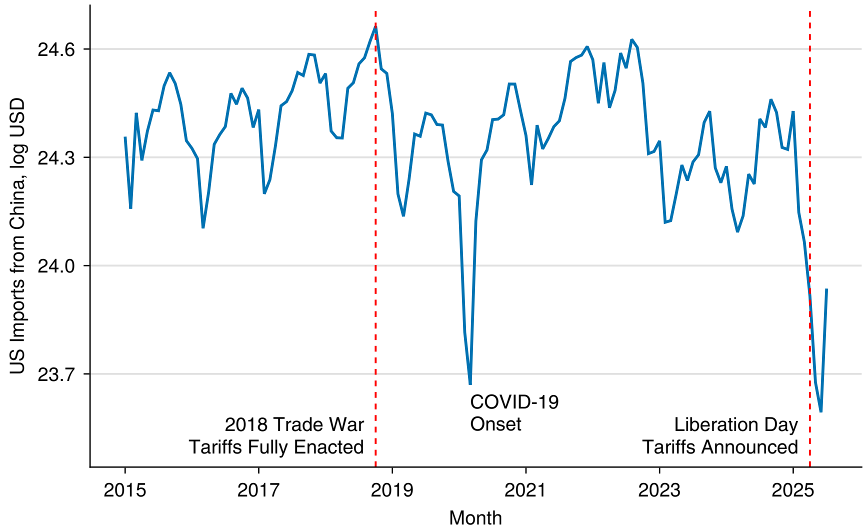

FIGURE 3: MONTHLY US TOTAL IMPORTS FROM CHINA, 2015–2025

Note: Data are obtained from the US Census Bureau and cover the last month of July 2025 from the most recent release at the time of writing.

impact on import flows. As we shall see shortly, inventory adjustments are likely to have played an important role in the drastic decline of the US imports from China.

B. *Anticipated 1-Month Tariff Hike*

The evolution of tariff rates in reality has been complicated. Yet, to deliver a clear demonstration on the model behavior enriched by the inventory adjustments, I abstract from many aspects of the reality and consider a simplified circumstance. Namely, starting from an initial steady state as in the economy calibrated as in Section III, I assume that there are anticipated tariff increases in the magnitude just as the original plan announced on April 2 that are anticipated by traders 3 months in advance. Without retaliation from any trade partner, I assume that these tariff increases are immediately reverted just after the first month they are implemented.

Figure 4a plots the model counterfactual outcomes for the impact of the anticipated tariff increases in terms of the percentage deviations from the initial steady-state level for the purchases of goods of US traders from domestic suppliers versus those from China. The latter can be interpreted as the model counterpart of US imports from China. In anticipation of future tariff increases, traders importing goods from China raise their order sizes before the tariff implementation and drastically cut down the order sizes during the month when the tariff rates are effective. In contrast, the US traders acquiring goods from domestic suppliers demonstrate much smaller responses. Figure 4b shows that the tariff impact is translated into very different welfare impact for US and China. There is some welfare loss for US started before the implementation of the tariffs measured in terms of real wage from aggregate labor income. The welfare loss only gradually disappears as time goes. With the specific specification of tariff shocks that are transitory, China even gains slightly after the tariff

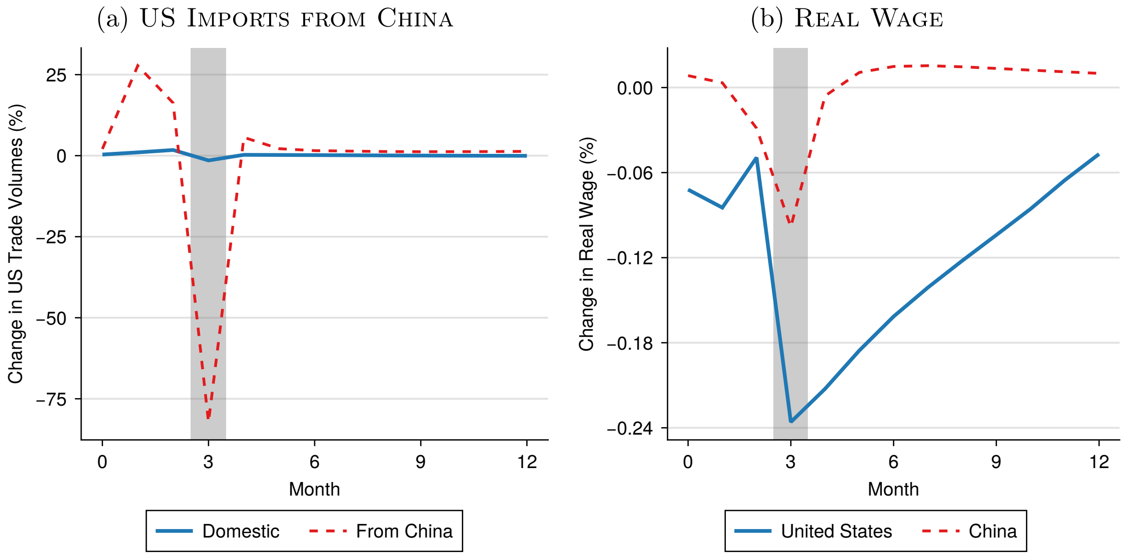

FIGURE 4: Impact of 1-Month Shocks Anticipated 3-Month Ahead

hike disappears, as the US importers start to restock their inventories with larger orders.

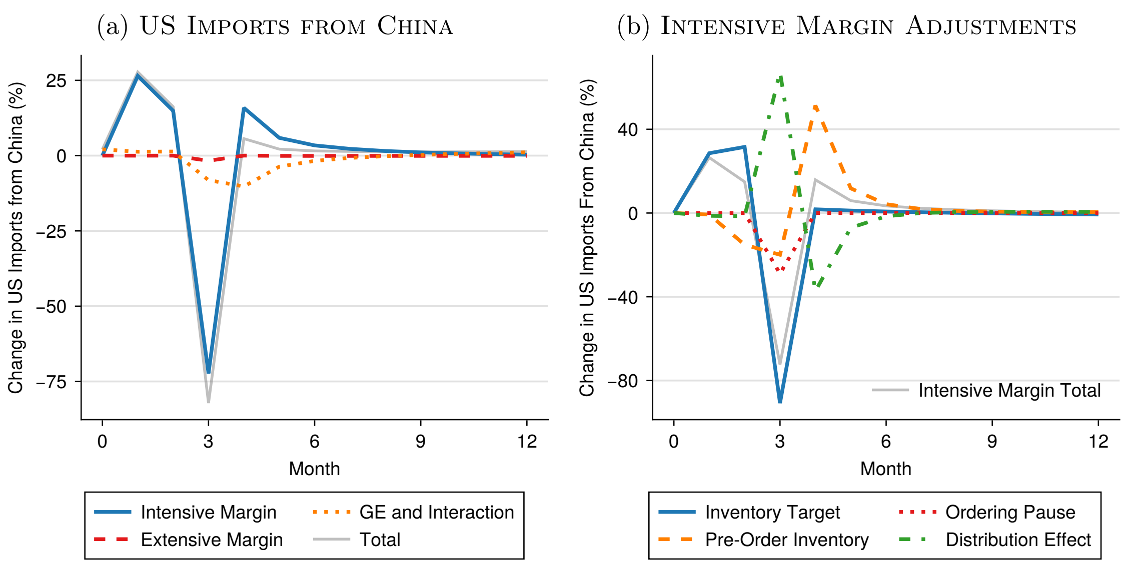

FIGURE 5: Decompositions of Responses of US Imports from China

To help understand the relative importance of model mechanisms behind the generated responses, I conduct decompositions described in Section II. Not surprisingly, Figure 5a shows that most of the responses of US imports from China arise from the intensive margin adjustments. That is, traders reduce their order sizes to a large extent and rely more on inventories to supply their domestic customers when the tariff increases are effective. The extensive margin adjustments are almost negligible in this specific scenario, because traders are aware of the immediate removal of the tariff rates. Since the sourcing decisions are based on future profit flows, most traders switching suppliers do not select an alternative sourcing country. As traders start to restock their reduced inventories after the tariff increases are reverted, there are some aggregate price increases that slightly dampen the restocking.

Figure 5b further decomposes the intensive margin responses into 4 sub-components. As expected, changes in inventory targets among the traders form the main driving force of the intensive margin adjustments. When the tariff increases are effective, traders with relatively high inventory stocks do not place an order. That is, the tariff increases are so large that the nonnegativity constraint for order size is binding for some traders. This ordering pause strengthens the decline in imports from China when the tariff increases are in place. On the other hand, more traders are holding inventories of higher vintages where order sizes are relatively larger, because of the ordering pause. The shift in distribution of traders across vintages results in some congestion as more traders with reduced inventories start to restock as the tariff hikes end.

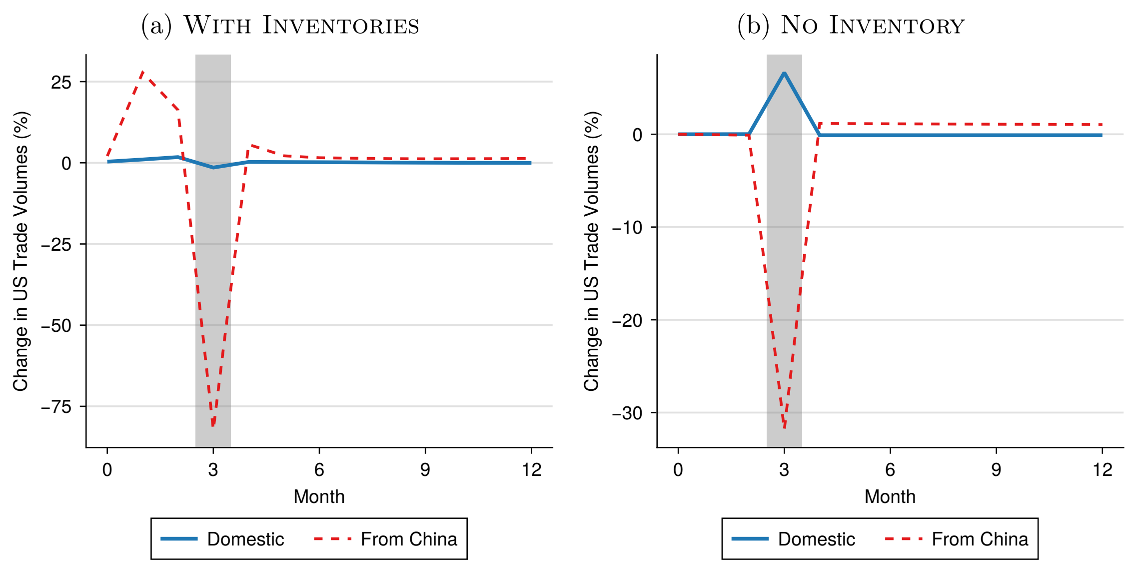

FIGURE 6: Impact of on US Imports from China

How has modeling inventories changed the counterfactual outcomes? Under the current model framework, having hazard rate of 1 in the period when traders just receive the goods removes the need of holding inventories completely. I therefore generate additional results based on an alternative model specification that only differs by setting all hazard rates to 1 to demonstrate the model outcomes when traders do not hold any inventory. Figure 6 contrasts the model outcomes on US imports. In the hypothetical scenario where traders do not hold inventory, there is virtually no dynamic impact on the trade flows. In fact, ordering of goods by traders and sales to domestic customers are now completely synchronized. The responses are driven by the familiar cross-sectional substitution under the CES aggregation, with expenditure shares move toward domestic varieties.

What about the welfare impact? Figure 7 contrasts the cumulative impact in real wage. Notably, under the baseline scenario with inventories, the welfare impact is much smaller than the scenario without inventory when the tariff increases are effective. This reflects the model characteristic that

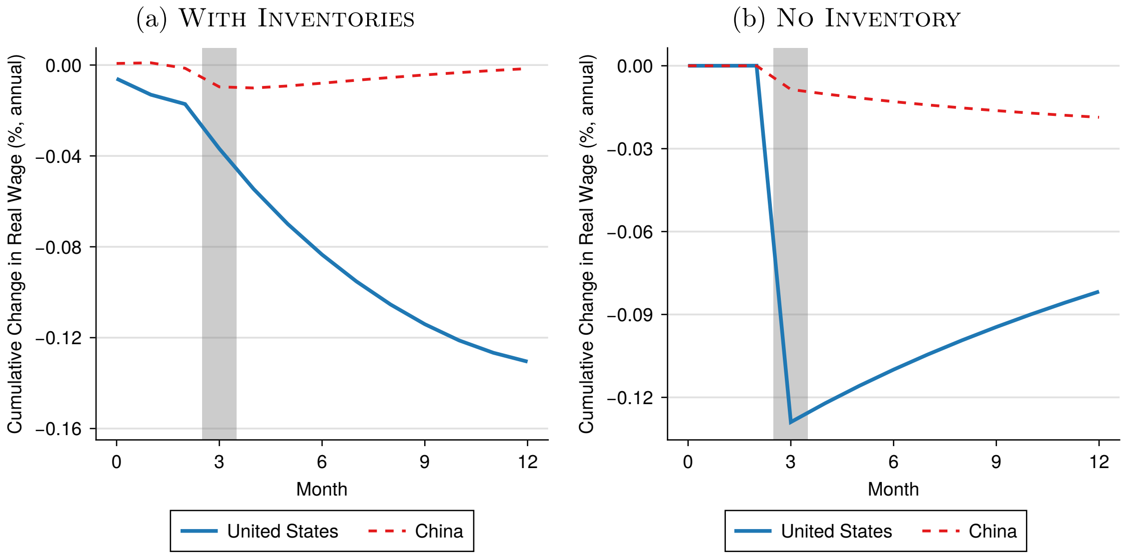

FIGURE 7: Cumulative Welfare Impact of Anticipated 1-Month Shocks

the tariff impact is not completely passed to the domestic customers when the tariffs are effective, because of the intertemporal pricing of the traders. However, as time passes, traders only gradually converge towards the initial steady state. There is continuing departure from the long-run optimal behavior. This results in sustained welfare loss. The cumulative welfare loss soon exceeds the counterpart under the scenario without inventory. Remarkably, despite the smaller welfare loss on impact, the overall welfare loss is even larger under the baseline scenario, because of the sustained disruptions across different time periods. This observation negates a conjecture that a world with inventories always suffers less from aggregate shocks. Intuitively, a world with and without inventories additionally differs by the risk faced by the traders that induces the holding of inventories in the first place. A complete answer on how modeling inventories affect the welfare implications cannot be provided without simultaneously addressing the changing motive of holding inventories.[35]

C. *Importance of Sectoral Linkages*

One of the key strengths of the model framework is its ability to accommodate detailed production networks, going well beyond the status quo of quantitative exercises involving inventories. It turns out that omitting the complex sectoral linkages existing in the real world can result in substantial differences on the model counterfactual outcomes. To illustrate, I consider an alternative specification where each economy consists of only 2 industries, a non-service industry and a service industry that are aggregated from the 34 industries considered under the baseline calibration. Only the non-service industry holds inventories and is exposed to tariff changes.

Figure 8 compares the responses of the US imports from China between the baseline specification

[35]The idea that comparing model outcomes with and without inventories requires addressing the endogenous motive of holding inventories has received attention during 1980s within the macroeconomic literature on inventories. Here, my observation under the international context echos this traditional thought.

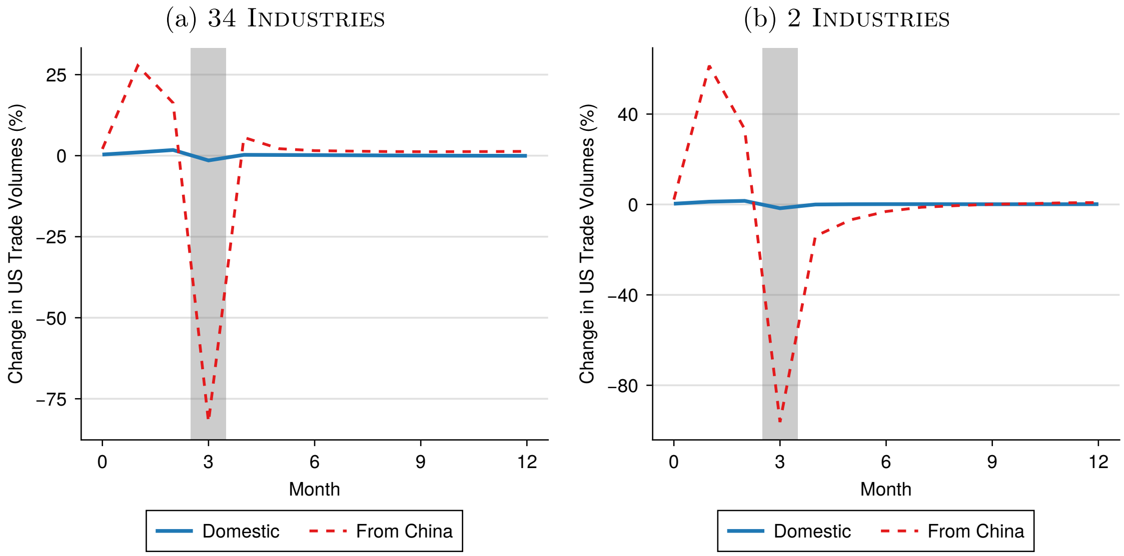

FIGURE 8: US Import Responses: Baseline vs 2-Industry Calibration

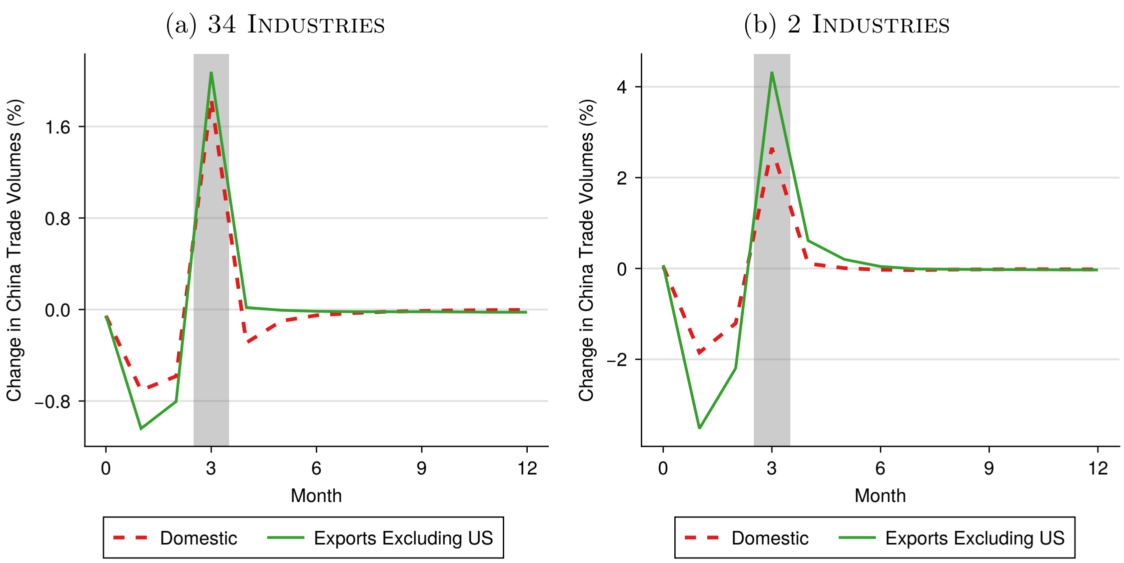

FIGURE 9: China Export Responses: Baseline vs 2-Industry Calibration

seen before and the results under the alternative specification with only 2 industries. With the highly simplified sectoral linkages, the US import flows demonstrate even stronger responses. Yet, the differences are more noticeable when considering trade flows not between US and China. For example, Figure 9 shows that the responses in exports from China to all countries except US would be more than doubled under the specification with only 2 industries.

Turning to the implications on the welfare impact, Figure C.2 in Appendix C shows that omitting the sectoral details would slightly exaggerate the impact on real wage for US. The more substantial differences, however, appear in the results for other economies. Figure 10 collects the cumulative real

(a) 34 Industries (b) 2 Industries

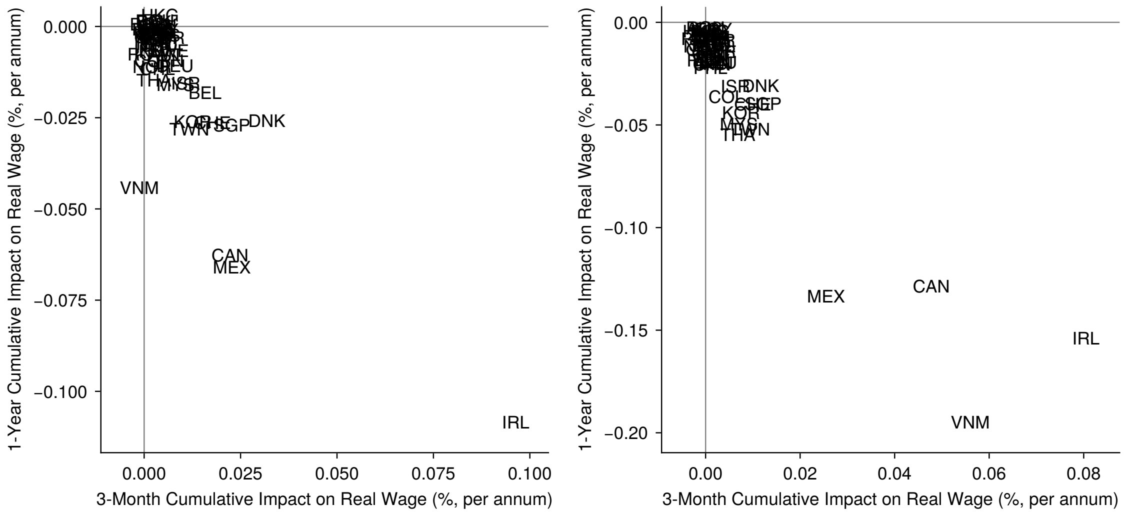

FIGURE 10: Cumulative Real Wage Impact across All Economies Except US

wage impact over the first 3 months and the first year across all economies excluding US. For most of the economies, the cumulative impact over the first year would be roughly doubled, if turning to the alternative specification with only 2 industries. The difference is particularly notable for Vietnam. With the baseline calibration, the welfare impact on Vietnam over the first three months is negligible and the one-year cumulative impact is under 0.05% at the annual rate. Yet, omitting the sectoral linkages results in a very different answer that Vietnam would gain by nearly 0.06% over the first 3 months and then revert to cumulative losses of nearly 0.2%. In Appendix C, I provide the results on nominal wage and aggregate price index for selected economies.

V. Conclusions

Addressing the apparent need of incorporating inventories into general equilibrium analysis of the tariff impact, I have developed a tractable dynamic Ricardian framework featuring two different margins of adjustments of trade with different relative importance depending on the nature of shocks. With the intensive margin adjustments, traders adjust their optimal inventory management by changing the size of the orders placed with their suppliers while taking into account the existing inventory stocks and future aggregate evolvement of the economy. With the extensive margin adjustments, traders occasionally switch to alternative suppliers that may be located in different countries based on the expected present value of future profit flows. The result of integrating both these adjustment margins in a unified framework is a model featuring rich dynamic behavior of traders that vary depending on both the time horizon under consideration and the magnitude and persistence of tariff impact.

To calibrate the model, I combine the detailed monthly US trade data at a disaggregated level with the nationally representative inventory data from the US national accounts. The model is calibrated to a realistic production network covering 34 industries in each of the 50 economies. Among the 34 industries, 19 of them hold inventories and are exposed to tariff changes. Despite the rich model features, solving the monthly general equilibrium counterfactual outcomes remain computationally tractable. Such a level of model details, which surpasses prior work by a substantial margin, is possible due to a novel technique leveraging a proportionality property of the solutions from the inventory optimization problem that has not been exploited in earlier work.

To demonstrate the model features in action, I construct simplified tariff shocks inspired by the 2025 Liberation Day tariffs. With the tariff increases that are large but known to be transient, the responses of the US import responses are dominated by the intensive margin adjustments. In particular, traders adjust their optimal inventory targets to stockpile goods in advance and run down inventories when the tariff increases are effective. This results in dynamic responses in trade flows that are detached from the expenditure variation among domestic customers. In contrast to a model where traders do not hold any inventory, the welfare loss measured in terms of changes in real wage is smaller on impact but persists across time. The overall cumulative welfare loss of the US economy remains substantial.

Furthermore, to highlight the strength of the model for accommodating realistic production networks, I compare the results with those from an alternative model specification where each economy consists of only 2 industries. It turns out that omitting sectoral linkages would exaggerate the model responses in both bilateral trade flows and welfare impact. In particular, the one-year cumulative real wage loss among countries other than US would be roughly doubled.

Despite only demonstrating quantitative results given a simplified path of tariff impact, I believe that the modeling framework has high potential for both addressing policy-relevant questions regarding the global economy and issues arising from other context as the unit of analysis does not have to be an industry in a country. Furthermore, the headroom left from the computational feasibility allows introducing model extensions that further augment the scope of analysis for future studies. It is the envision of the author that dynamic general equilibrium analysis involving inventory decisions of individual economic agents will become more widely applicable with the advances in modeling techniques.

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Appendix A. Model Extensions

A. *Non-Credible Tariff Policy*

In reality, traders may face substantial uncertainty in predicting the future changes of tariff rates. Yet, the optimal responses in the model require traders taking into account the future changes including tariff rate changes. The uncertainty existing in real-world scenarios would therefore be ignored if we assume that the traders are fully aware of the actual future tariff rate changes. To address this departure from the reality, I introduce a notion of non-credible tariff policy into the model. That is, traders do not view tariff rate changes as permanent no matter how they are communicated. Instead, traders assess the likelihood for the new tariff rates to persist based on their own beliefs. The model presented in Section I is therefore augmented to allow individual beliefs of the traders that may differ from the actual path of tariff rates. For later discussions, this augmented model is referred to as the model economy under non-credible tariff policy.

Tariff Changes.—All tariff policies are announced with a set of precise tariff rate changes and their initial effective dates. However, there is no guarantee as to how long these tariffs will persist after taking effect, except for the immediate next period.[36] It is entirely up to the policymakers to revise these tariff rates again in the future. Suppose the initial tariff rate is τ_{sdi0}. A bilateral tariff change for a given industry is formally defined as the following process

$$\tau_{sdit} = \tau_{sdi0} + \Delta\tau_{sdi}\xi^{\tau}_{sdit}$$

where $\Delta\tau_{sdi}$ is the change in tariff rate and ξ^{τ}_{sdit} is a random variable following a Bernoulli distribution. The parameter $\chi^{\tau}_{sdit} \equiv \Pr(\xi^{\tau}_{sdit} = 1)$ captures the unobserved potential for the policymakers to revert the tariff changes. Notice that a permanent change in tariff rate is a special case in which $\chi^{\tau}_{sdit} = 1$.

Bayesian Learning.—Importers never observe the true parameter χ^{τ}_{sdit}.[37] Instead, they conduct sequential Bayesian inference on the (subjective) distribution of χ^{τ}_{sdit} in every period since the tariff change takes effect. For simplicity, I assume that all traders form the same prior belief that

$$\chi^{\tau}_{sdit+2} \sim \text{Beta}(\alpha^{\chi}, \beta^{\chi})$$

when a tariff change is announced but is not going to take effect at least for the next period.[38] In the special case where $\alpha^{\chi} = \beta^{\chi} = 1$, the distribution is uniform over the unit interval, meaning that traders have no idea on how likely the tariff rate will be reverted. On the other hand, as α^{χ} or β^{χ}

[36] At the time when traders determine the order sizes, there is no uncertainty regarding whether the tariff rate is going to be changed for the next period. This assumption prevents the traders from discounting the tariff rate changes that are immediately taking effect.

[37] Even when the policymakers claim the tariff changes are permanent, traders would not interpret that as a signal of $\chi^{\tau}_{sdit} = 1$.

[38] Recall that it is assumed that the tariff rate for the next period, the one that directly affects the order size set today, cannot be changed. It is therefore the tariff rate in the period after that which remains uncertain.

goes to infinity, traders form a sharp belief on the magnitude of χ^{τ}_{sdit+2}.[39] As time goes, with the tariff change $\Delta\tau_{sdi}$ being in place for T periods, traders make decisions based on the belief updated with Bayes' rule

$$\chi^{\tau}_{sdit+2} \,\Big|\, \left\{\xi^{\tau}_{sdit-T+s+1} = 1 \text{ for all } 1 \leqslant s \leqslant T\right\} \sim \text{Beta}(\alpha^{\chi} + T, \beta^{\chi}).$$

Therefore, as the new tariff rate remains effective, traders gradually increase the perceived likelihood that the new rate will continue being relevant in the future. In particular, as T goes to infinity, the updated belief on χ^{τ}_{sdit+2} converges to a point mass with $\Pr(\chi^{\tau}_{sdit+2} = 1) = 1$. That is, traders eventually get convinced that the tariff rate change is permanent if it is never revised again. Let $F_{\chi^{\tau}_{sdit+2}}$ denote the distribution function that characterizes the current belief on χ^{τ}_{sdit+2}. The value function from Equation (1) is augmented to

$$V^{s}_{dit}(h_{\omega t}, n_{\omega t}; \xi_{\omega t}, F_{\chi^{\tau}_{sdit+2}}) = \max_{n_{\omega t+1} \geqslant 0,\ p_{\omega t}} \pi_t(p_{\omega t}) + \frac{1}{1+i_t} \int \mathbb{E}_t \left[V^{s}_{dit+1}(h_{\omega t+1}, n_{\omega t+1}; \xi_{\omega t+1}) \mid \chi^{\tau}_{sdit+2} \right] \mathrm{d}F_{\chi^{\tau}_{sdit+2}}$$

where the belief on the future possibility for the tariff change to be reverted is taken into account with the integral over the conditional expectation.

Appendix B. Additional Details on Model Calibration

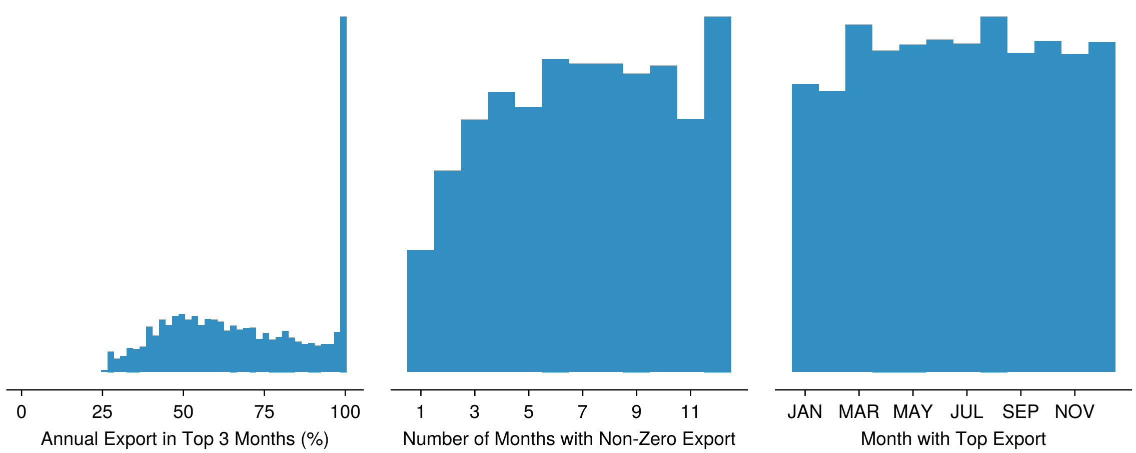

FIGURE B.1: Histograms for the Selected US Export Sample

[39]The duration of the new tariff rate remains uncertain even when the belief is a point mass on a specific level of χ^{τ}_{sdit+2} such that $0 < \chi^{\tau}_{sdit+2} < 1$. However, if $\chi^{\tau}_{sdit+2} = 1$, traders always view the tariff change as permanent.

TABLE B.1: BEA NIPA Series of Real Inventories Selected for Calibration

Model Industry	NIPA Series Code	NIPA NAICS Industry
1	KS4225	Farm product raw material wholesalers
2	KS4215	Metal and mineral (except petroleum) wholesalers
3	KS311	Food manufacturing
	KS312	Beverage and tobacco product manufacturing
	KS4224	Grocery and related products wholesalers
	N239RX	Food and beverage stores
4	KS313	Textile mills
	KS314	Textile product mills
	KS315	Apparel manufacturing
	KS316	Leather and allied product manufacturing
	KS4223	Apparel, piece goods, and notions wholesalers
	KS448	Clothing and clothing accessories stores
5	KS321	Wood product manufacturing
	KS4213	Lumber and other construction materials wholesalers
6	KS322	Paper manufacturing
	KS323	Printing and related support activities
	KS4221	Paper and paper products wholesalers
7	KS324	Petroleum and coal product manufacturing
	KS4227	Petroleum and petroleum products wholesalers
8	KS325	Chemical manufacturing
	KS4226	Chemical and allied products wholesalers
9	KS4222	Drugs and druggists' sundries wholesalers
10	KS326	Plastics and rubber product manufacturing
11	KS327	Nonmetallic mineral product manufacturing
	KS4215	Metal and mineral (except petroleum) wholesalers
12	KS331	Primary metal manufacturing
	KS4215	Metal and mineral (except petroleum) wholesalers
13	KS332	Fabricated metal product manufacturing
	KS4215	Metal and mineral (except petroleum) wholesalers
14	KS334	Computer and electronic product manufacturing
	KS4243	Computers and software wholesalers
15	KS335	Electrical equipment, appliance, and component manufacturing
	KS4216	Electrical goods wholesalers
16	KS333	Machinery manufacturing
	KS4218	Machinery, equipment, and supplies wholesalers
17	KS3MV	Motor vehicle and parts manufacturing
	KS4211	Motor vehicles, parts, and supplies wholesalers
	N864RX	Motor vehicle and parts dealers
18	KS3OT	Other transportation equipment manufacturing
19	KS337	Furniture and related product manufacturing
	KS4212	Furniture and home furnishings wholesalers

Notes: In the model, 19 out of 34 industries hold inventories. See Table 1 for their names.

TABLE B.2: PARAMETERS OF TRADE ELASTICITIES FOR EACH INDUSTRY

Industry in Model	Long-Run Trade Elasticity (θ)	Short-Run Trade Elasticity ($\sigma - 1$)	Annual Rate of Supplier Switching (ζ)
Agriculture	5.69	3.40	0.18
Mining	1.58	0.23	0.73
Food	5.69	3.40	0.18
Textiles	5.69	3.40	0.18
Wood	2.01	0.51	0.09
Paper and printing	2.01	0.51	0.09
Coke and petroleum	2.01	0.51	0.09
Chemicals	2.32	0.74	0.11
Pharmaceuticals	2.32	0.74	0.11
Rubber and plastics	2.01	0.51	0.09
Non-metallic minerals	6.55	0.66	0.08
Basic metals	3.33	0.40	0.07
Fabricated metals	3.33	0.40	0.07
Computer	5.64	0.70	0.10
Electrical equipment	1.58	0.23	0.73
Machinery	3.33	0.40	0.07
Motor vehicles	3.33	0.40	0.07
Other transport equipment	3.33	0.40	0.07
Furniture and others	1.58	0.23	0.73
All 15 service industries	5.48	0.79	0.06

Notes: Estimates are obtained from Chen et al. (2025) where the same set of 34 industries are considered. Some estimates are shared across multiple industries due to the availability of IV. Estimation is based on nonlinear GMM estimation with bilateral trade data at the 6-digit HS code level and plausibly exogenous tariff variation identified by Boehm, Levchenko and Pandalai-Nayar (2023).

Appendix C. Additional Results from the Quantitative Applications

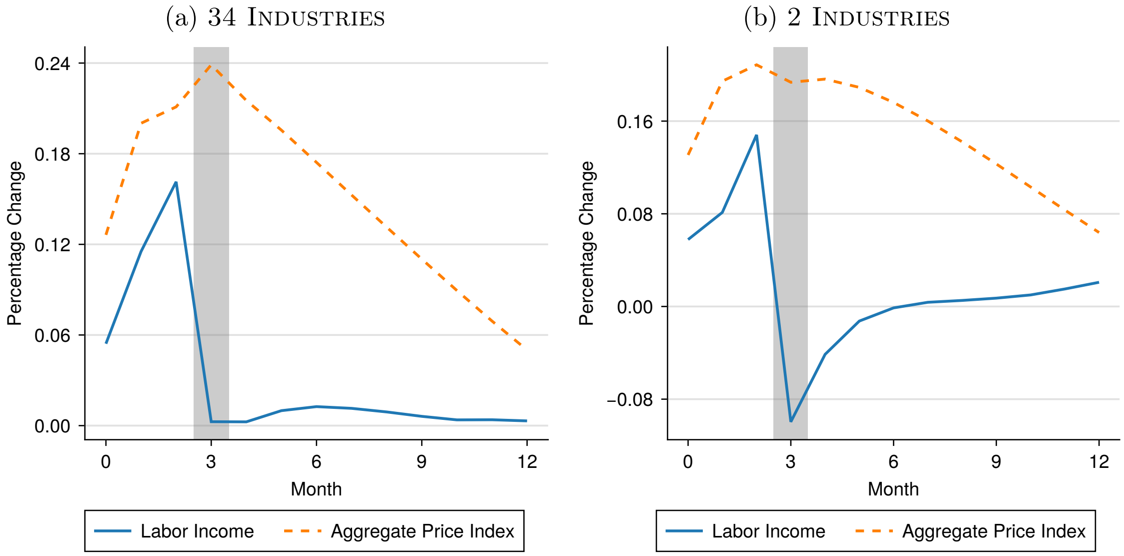

FIGURE C.1: Responses of Labor Income and Aggregate Price in US

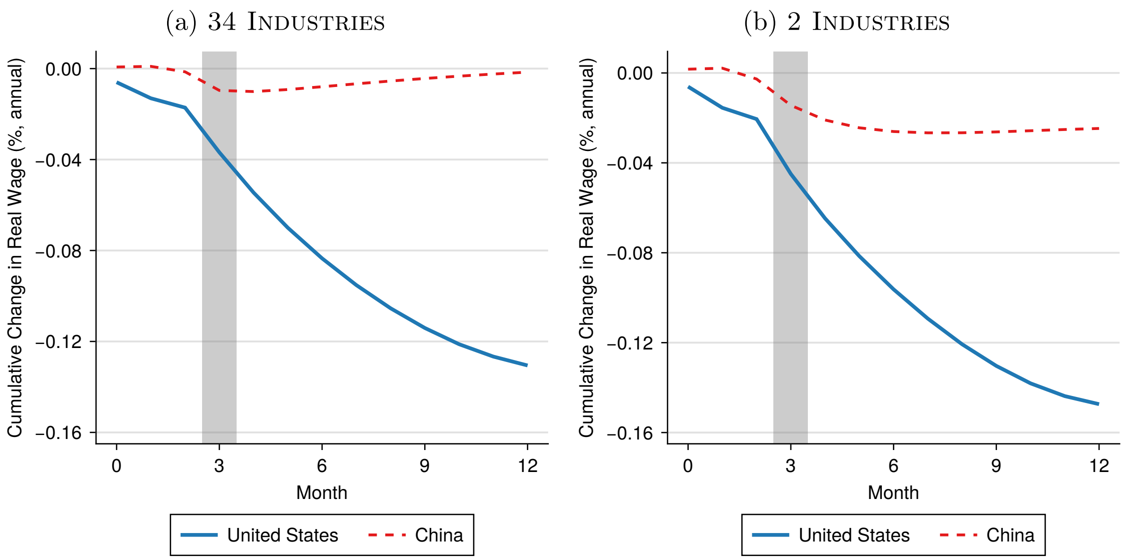

FIGURE C.2: Cumulative Real Wage Impact: Baseline vs 2-Industry Calibration

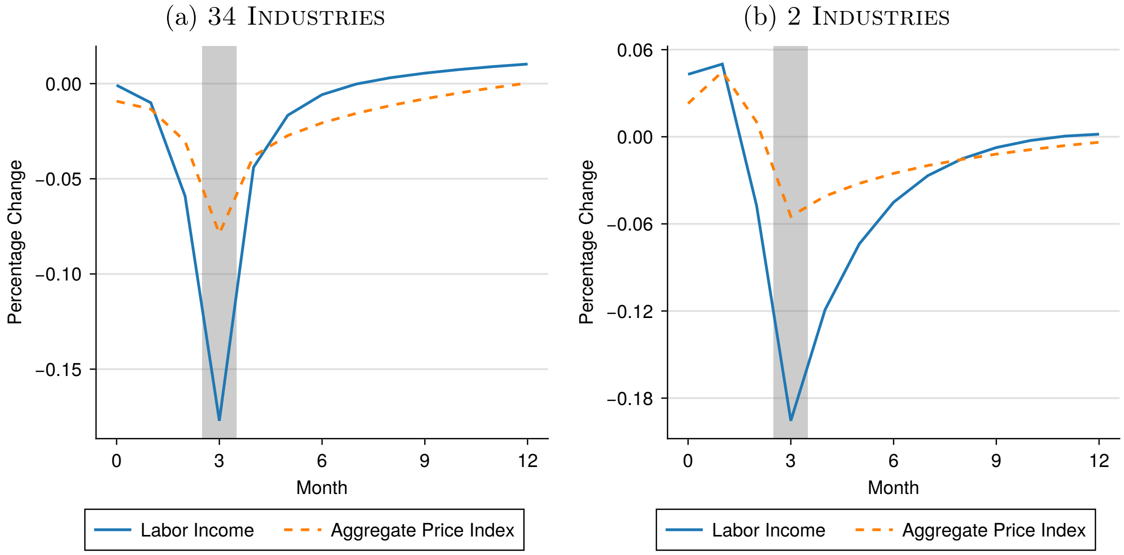

FIGURE C.3: Responses of Labor Income and Aggregate Price in China

FIGURE C.4: Responses of Labor Income and Aggregate Price in Vietnam

Appendix D. Sketch of Solution Algorithm

Most of the variables in model are represented as the ratios over the initial steady-state level. A sufficiently large number of time periods T is selected so that in the last period T, the economy is sufficiently close to a steady state. A reasonably large maximum vintage A is chosen so that with inventories of vintage A, the hazard rate of receiving the goods in the next period is 1.

Given a path of changes in tariffs $\{\hat{\tau}_{sdit}\}_{t=0}^{T}$ from time 0 to T, the counterfactual outcomes are obtained by solving a nested system of nonlinear equations that determine 6 sets of equilibrium paths: (1) the country-industry-level total expenditure $\{X_{djt}\}_{t=0}^{T}$; (2) the changes in country-industry-level expected present value of profit flows of just-switching traders $\{\hat{V}_{sdit}\}_{t=0}^{T}$; (3) the optimal inventory target of a benchmark trader in each country-industry; (4) the changes in prices set by traders that have not received new orders since the announcement of shocks; (5) the changes in country-level wages $\{\hat{w}_{dt}\}_{t=0}^{T}$; and (6) the changes in country-industry-level price indices faced by producers $\{\hat{P}_{djt}\}_{t=0}^{T}$. The computational algorithm is outlined below.

Step W0 Guess changes in factor prices $\{\hat{w}_{dt}\}_{t=0}^{T}$ and price indices $\{\hat{P}_{djt}\}_{t=0}^{T}$.

Step W1 For each period $t = 0, \ldots, T$, iterate through the V steps:

- **Step V0** Guess changes in $\{\hat{V}_{sdit}\}_{t=0}^{T}$, optimal inventory targets and the prices set by benchmark traders not yet receiving goods after the announcement of shocks.
- **Step V1** Update sourcing decisions among just-switching traders.
- **Step V2** Update optimal prices set by all benchmark traders by vintage and cohort.
- **Step V3** Update order sizes for all benchmark traders given optimal inventory targets and existing inventory levels.
- **Step V4** Update the transition rates across vintages based on whether the order size placed by the benchmark trader with each vintage of inventories is positive.
- **Step V5** Update expenditure shares on varieties across vintages within each cohort of traders.
- **Step V6** Update expenditure shares on varieties across cohorts of traders.
- **Step V7** Update expenditure shares by sourcing countries within each country-industry.
- **Step V8** Update aggregate imports and current profit flows of the benchmark traders.
- **Step V9** Update production, tariff revenue, aggregate trader profits.
- **Step V10** Solve the aggregate expenditures X_{djt} on goods from each industry-country.
- **Step V11** Update the inventory levels of the benchmark traders for the next period.
- **Step V12** If $t = T$, update $\{\hat{V}_{sdit}\}_{t=0}^{T}$, optimal inventory targets and the prices set by benchmark traders not yet receiving goods after the announcement of shocks.

Step W2 Return to **Step W1** and repeat until the paths are sufficiently close across the guesses.

Step W3 Update changes in factor prices $\{\hat{w}_{dt}\}_{t=0}^{T}$ and price indices $\{\hat{P}_{djt}\}_{t=0}^{T}$.

Step W4 Return to **Step W0** and repeat until the paths are sufficiently close across the guesses.